

An assumption is that all heterogeneous agents are normally distributed, and are independent of each other.

1 PRODUCER OF GOODS

If producers have heterogeneous technologies and have bounded rationality, their present value gets modified to the following expression:

$$V(0) = \int_0^{\infty} [\alpha_i p_{1i}(t) F_i(K(t), L(t)) - p_{2i}(t) I(t) - p_{3i}(t) L(t)] e^{-r_i t} dt \quad (50)$$

Due to imperfect information, $p_{1i}(t)$, $p_{2i}(t)$, and $p_{3i}(t)$ are prices known to producer which are assumed to be positive functions of actual market prices. This implies if $p_1(t)$, $p_2(t)$, and $p_3(t)$ increase, $p_{1i}(t)$, $p_{2i}(t)$, and $p_{3i}(t)$ also increase respectively. α_i is fraction of market price charged by producer to middleman; r_i is discount rate; $L(t)$ (labor) and $I(t)$ (level of investment) are *control variables* and $K(t)$ is *state variable*. Maximization problem is as under:

$$\underset{\{L(t), I(t)\}}{\text{Max}} V(0) = \int_0^{\infty} [\alpha_i p_{1i}(t) F_i(K(t), L(t)) - p_{2i}(t) I(t) - p_{3i}(t) L(t)] e^{-r_i t} dt,$$

subject to following constraints:

$\dot{K}(t) = I(t) - \delta_i K(t)$ (state equation, describing how state variable changes with time),

$K(0) = K_0$ (initial condition),

$K(t) \geq 0$ (non-negativity constraint on state variable),

$K(\infty)$ free (terminal condition).

Current-value Hamiltonian is as under:

$$\tilde{H} = \alpha_i p_{1i}(t) F_i(K(t), L(t)) - p_{2i}(t) I(t) - p_{3i}(t) L(t) + \mu(t) [I(t) - \delta_i K(t)] \quad (51)$$

Maximizing conditions are as under:

(i) $L^*(t)$ and $I^*(t)$ maximize \tilde{H} for all t : $\frac{\partial \tilde{H}}{\partial L} = 0$ and $\frac{\partial \tilde{H}}{\partial I} = 0$,

(ii) $\dot{\mu} - r_i \mu = -\frac{\partial \tilde{H}}{\partial K}$,

(iii) $\dot{K}^* = \frac{\partial \tilde{H}}{\partial \mu}$ (this just gives back the state equation),

(iv) $\lim_{t \rightarrow \infty} \mu(t) K(t) e^{-r_i t} = 0$ (the transversality condition).

Conditions (i) and (ii) can be expressed as follows:

$$\frac{\partial \tilde{H}}{\partial L} = \alpha_i p_{1i}(t) F_{i2}'(K(t), L(t)) - p_{3i}(t) = 0 \quad (52)$$

$$\frac{\partial \tilde{H}}{\partial I} = -p_{2i}(t) + \mu(t) = 0 \quad (53)$$

And

$$\dot{\mu} - r_i \mu = -\frac{\partial \tilde{H}}{\partial K} = -[\alpha_i p_{1i}(t) F_{i1}'(K(t), L(t)) - \delta_i \mu(t)] \quad (54)$$

Substituting values of $\dot{\mu}$ and μ from Equation (53) in (54) yields

$$\alpha_i p_{1i}(t) F_{i1}'(K(t), L(t)) - (r_i + \delta_i) p_{2i}(t) + \dot{p}_{2i}(t) = 0$$

If $p_1(t)$ (price of goods) increases, producer faces following inequalities at existing level of investment and labor:

$$\alpha_i p_{1i}(t) F'_{i2}(K(t), L(t)) - p_{3i}(t) > 0,$$

$$\alpha_i p_{1i}(t) F'_{i1}(K(t), L(t)) - (r_i + \delta_i) p_{2i}(t) + \dot{p}_{2i}(t) > 0$$

Similarly, if either inventory of capital or labor goes up (ceteris paribus), producer faces following inequalities:

$$\alpha_i p_{1i}(t) F'_{i2}(K(t), L(t)) - p_{3i}(t) > 0, \alpha_i p_{1i}(t) F'_{i1}(K(t), L(t)) - (r_i + \delta_i) p_{2i}(t) + \dot{p}_{2i}(t) > 0$$

Therefore, if price of goods, inventory of capital or labor goes up (ceteris paribus), producer will increase production as follows:

$$W_{s1i} = -K_{s11i} \varepsilon_1(t - \tau_{s11i}) + U_{s12i} M_2(t - \tau_{s12i}) + K_{s13i} M_3(t - \tau_{s13i}),$$

which implies average production will increase as follows:

$$\begin{aligned} \frac{1}{N} \sum_{i=1}^N W_{s1i}(\text{Goods}) = & -\frac{1}{N} \sum_{i=1}^N K_{s11i} \varepsilon_1(t - \frac{1}{N} \sum_{i=1}^N \tau_{s11i}) + \frac{1}{N} \sum_{i=1}^N U_{s12i} M_2(t - \frac{1}{N} \sum_{i=1}^N \tau_{s12i}) \\ & + \frac{1}{N} \sum_{i=1}^N K_{s13i} M_3(t - \frac{1}{N} \sum_{i=1}^N \tau_{s13i}), \end{aligned}$$

Or

$$W_{s1}(\text{Goods}) = -K_{s11} \varepsilon_1(t - \tau_{s11}) + K_{s12} f_{s12} \kappa M_2(t - \tau_{s12}) + K_{s13} M_3(t - \tau_{s13}) \quad (55)$$

Where

$W_{s1} = \frac{1}{N} \sum_{i=1}^N W_{s1i}$, $K_{s11} = \frac{1}{N} \sum_{i=1}^N K_{s11i}$, $K_{s12} f_{s12} \kappa = \frac{1}{N} \sum_{i=1}^N U_{s12i}$, $\tau_{s11} = \frac{1}{N} \sum_{i=1}^N \tau_{s11i}$, $\tau_{s12} = \frac{1}{N} \sum_{i=1}^N \tau_{s12i}$, $\tau_{s13} = \frac{1}{N} \sum_{i=1}^N \tau_{s13i}$, i.e., each parameter is an average of parameters of N number of producers. As all these parameters are to be estimated empirically from national level practical data, it will capture practical behavior of agents; and hence heterogeneity of firms and bounded rationality when present in real world will get captured during estimation.

2 CONSUMER OF GOODS

If consumers have heterogeneous preferences and have bounded rationality, their present value gets modified to following expression:

$$V(0) = \int_0^\infty U_i(x_i(t)) e^{-\rho_i t} dt \quad (56)$$

ρ_i being discount rate; and $x_i(t)$ (consumption) as *control variable*. Maximization problem is as under:

$$\text{Max}_{\{x(t)\}} V(0) = \int_0^\infty U_i(x_i(t)) e^{-\rho_i t} dt,$$

subject to following constraints:

$\dot{a}_i(t) = \beta_i p_{2i}(t) a_i(t) + p_{3i}(t) L_i(t) - p_{1i}(t) x_i(t)$ (state equation, describing how state variable changes with time). β_i is fraction of $p_{2i}(t)$ charged by households to financial intermediaries, $a_i(t)$ is asset holdings (a *state variable*) and $p_{3i}(t)$ and $p_{2i}(t)$ are time path of wages and return on assets.

$a_i(0) = a_{is}$ (initial condition),

$a_i(t) \geq 0$ (non-negativity constraint on state variable),

$a_i(\infty)$ free (terminal condition).

Current-value Hamiltonian is as under:

$$\tilde{H} = U_i(x_i(t)) + \mu(t)[\beta_i p_{2i}(t)a_i(t) + p_{3i}(t)L_i(t) - p_{1i}(t)x_i(t)] \quad (57)$$

Maximizing conditions are as under:

(i) $x_i^*(t)$ maximizes \tilde{H} for all t : $\frac{\partial \tilde{H}}{\partial x_i} = 0$,

(ii) $\dot{\mu} - \rho_i \mu = -\frac{\partial \tilde{H}}{\partial a_i}$,

(iii) $\dot{a}_i^* = \frac{\partial H}{\partial \mu}$ (this just gives back the state equation),

(iv) $\lim_{t \rightarrow \infty} \mu(t)a_i(t)e^{-\rho_i t} = 0$ (the transversality condition).

Conditions (i) and (ii) can be expressed as follows:

$$\frac{\partial \tilde{H}}{\partial x_i} = U_i'(x_i(t)) - \mu(t)p_{1i}(t) = 0 \quad (58)$$

And

$$\dot{\mu} - \rho_i \mu = -\frac{\partial \tilde{H}}{\partial a_i} = -\mu(t)\beta_i p_{2i}(t) \quad (59)$$

Due to imperfect information, $p_{1i}(t)$, and $p_{2i}(t)$ are prices known to i th consumer which are assumed to be positive functions of actual market prices. This implies if $p_1(t)$, and $p_2(t)$ increase, $p_{1i}(t)$, and $p_{2i}(t)$ also increase respectively. If price of good x goes up, consumer faces (at previous level of consumption) following inequality:

$$\frac{\partial \tilde{H}}{\partial x_i} = U_i'(x_i(t)) - \mu(t)p_{1i}(t) < 0$$

To satisfy condition of dynamic optimization after price increase, consumer will decrease consumption of good x . If inventory of unsold labor goes up, production of x by producer goes up which brings price of x down after a time delay, and hence consumption of x increases. If inventory of money/capital goes up, price of capital goes down and consumer faces following inequality:

$$-\frac{\partial \tilde{H}}{\partial a_i} = -\mu(t)\beta_i p_{2i}(t) > \dot{\mu} - \rho_i \mu,$$

which implies consumer will reduce purchase of assets and will increase consumption of $x(t)$. Above discussion implies,

$$W_{d1i}(\text{Goods}) = -K_{d11i}P_1 + U_{d121i}M_2(t) + U_{d122i}M_2(t) + K_{d13i}M_3(t - \tau_{d13}),$$

$$W_{d1i} = \text{Change in consumption of good } x \text{ by } i\text{th consumer},$$

which implies average consumption will increase as follows:

$$\begin{aligned} \frac{1}{N} \sum_{i=1}^N W_{d1i}(\text{Goods}) = & -\frac{1}{N} \sum_{i=1}^N K_{d11i}P_1 + \frac{1}{N} \sum_{i=1}^N U_{d121i}M_2(t) + \frac{1}{N} \sum_{i=1}^N U_{d122i}M_2(t) + \\ & \frac{1}{N} \sum_{i=1}^N K_{d13i}M_3(t - \frac{1}{N} \sum_{i=1}^N \tau_{d13i}), \end{aligned}$$

Or

$$W_{d1}(\text{Goods}) = -K_{d11}P_1 + K_{d121}f_{d12}\kappa M_2(t) + K_{d122}(1 - \kappa)M_2(t) + K_{d13}M_3(t - \tau_{d13}) \quad (60)$$

where $W_{d1} = \frac{1}{N} \sum_{i=1}^N W_{d1i}$, $K_{d11} = \frac{1}{N} \sum_{i=1}^N K_{d11i}$, $K_{d121}f_{d12}\kappa = \frac{1}{N} \sum_{i=1}^N U_{d121i}$, $K_{d122}(1 - \kappa) = \frac{1}{N} \sum_{i=1}^N U_{d122i}$, $\tau_{d13} = \frac{1}{N} \sum_{i=1}^N \tau_{d13i}$, i.e., each parameter is an average of parameters of N number of consumers. As all these parameters are to be estimated empirically from national level practical data, it will capture practical behavior of agents; and hence heterogeneity of consumers and bounded rationality when present in real world will get captured during estimation.

3 HOUSEHOLD/PRODUCER OF FUNDS

With heterogeneous preferences and bounded rationality, from eq. (59), if price of capital/interest rate goes up, i th household faces following inequality:

$$-\frac{\partial \tilde{H}}{\partial a_i} = -\mu(t)\beta_i p_{2i}(t) < \dot{\mu} - \rho_i \mu,$$

which implies after an interest rate increase, i th household increases production/supply of funds to financial intermediary. If inventory of goods and unsold labor goes up (which increases production of good x after a time delay), price of good x goes down, and household faces following expression:

$$\frac{\partial \tilde{H}}{\partial x_i} = U_i'(x_i(t)) - \mu(t)p_{1i}(t) > 0$$

and household will increase consumption of good x with fewer resources left for bank deposits which will go down. Above discussion implies,

$$W_{s2i}(\text{Money/Capital}) = -K_{s21i}M_1 - K_{s22i}\varepsilon_2(t - \tau_{s22i}) + K_{s23i}M_3(t - \tau_{s23i}),$$

which implies average production will increase as follows:

$$\begin{aligned} \frac{1}{N} \sum_{i=1}^N W_{s2i}(\text{Money/Capital}) &= -\frac{1}{N} \sum_{i=1}^N K_{s21i}M_1 - \frac{1}{N} \sum_{i=1}^N K_{s22i}\varepsilon_2(t - \tau_{s22i}) + \\ &\quad \frac{1}{N} \sum_{i=1}^N K_{s23i}M_3(t - \tau_{s23i}), \end{aligned}$$

Or

$$W_{s2}(\text{Money/Capital}) = -K_{s21}M_1 - K_{s22}\varepsilon_2(t - \tau_{s22}) + K_{s23}M_3(t - \tau_{s23}) \quad (61)$$

Where

$W_{s2} = \frac{1}{N} \sum_{i=1}^N W_{s2i}$, $K_{s21} = \frac{1}{N} \sum_{i=1}^N K_{s21i}$, $K_{s22} = \frac{1}{N} \sum_{i=1}^N K_{s22i}$, $K_{s23} = \frac{1}{N} \sum_{i=1}^N K_{s23i}$, $\tau_{s22} = \frac{1}{N} \sum_{i=1}^N \tau_{s22i}$, $\tau_{s23} = \frac{1}{N} \sum_{i=1}^N \tau_{s23i}$, i.e., each parameter is an average of parameters of N number of households/producers of funds. As all these parameters are to be estimated empirically from national level practical data, it will capture practical behavior of agents; and hence heterogeneity of households and bounded rationality when present in real world will get captured during estimation.

Firm/Consumer of Funds

With heterogeneous preferences and bounded rationality, from eq. (9), if price of capital/interest rate goes up, *ith* firm faces following inequality:

$$\alpha_i p_{1i}(t) F'_{i1}(K(t), L(t)) - (r_i + \delta_i) p_{2i}(t) + \dot{p}_{2i}(t) < 0,$$

which implies after an interest rate increase, *ith* consumer of funds demands lower amount of funds/capital. If inventory of goods goes up, $p_{1i}(t)$ decreases, and producer faces following inequalities at existing level of investment and labor:

$$\alpha_i p_{1i}(t) F'_{i2}(K(t), L(t)) - p_{3i}(t) < 0,$$

$$\alpha_i p_{1i}(t) F'_{i1}(K(t), L(t)) - (r_i + \delta_i) p_{2i}(t) + \dot{p}_{2i}(t) < 0$$

This implies that after an increase in goods inventory, producer demands a lower quantity of capital. Similarly, if inventory of unsold labor goes up, labor becomes cheaper, and demand of capital (which compliments labor) by firm goes up due to following inequality faced by firm

$$\alpha_i p_{1i}(t) F'_{i2}(K(t), L(t)) - p_{3i}(t) > 0$$

Above discussion implies the following expression:

$$W_{d2i}(\text{Money/Capital}) = -K_{d21i}M_1(t - \tau_{d21i}) - K_{d22i}P_2 + K_{d23i}M_3(t - \tau_{d23i}),$$

which implies average demand will change as follows:

$$\begin{aligned} \frac{1}{N} \sum_{i=1}^N W_{d2i}(\text{Money/Capital}) &= -\frac{1}{N} \sum_{i=1}^N K_{d21i}M_1(t - \frac{1}{N} \sum_{i=1}^N \tau_{d21i}) - \frac{1}{N} \sum_{i=1}^N K_{d22i}P_2 + \\ &\quad \frac{1}{N} \sum_{i=1}^N K_{d23i}M_3(t - \frac{1}{N} \sum_{i=1}^N \tau_{d23i}), \end{aligned}$$

Or

$$W_{d2}(\text{Money/Capital}) = -K_{d21}M_1(t - \tau_{d21}) - K_{d22}P_2 + K_{d23}M_3(t - \tau_{d23}) \quad (62)$$

Where

$$W_{d2} = \frac{1}{N} \sum_{i=1}^N W_{d2i}, K_{d21} = \frac{1}{N} \sum_{i=1}^N K_{d21i}, K_{d22} = \frac{1}{N} \sum_{i=1}^N K_{d22i}, K_{d23} = \frac{1}{N} \sum_{i=1}^N K_{d23i}, \tau_{d21} = \frac{1}{N} \sum_{i=1}^N \tau_{d21i}, \tau_{d23} = \frac{1}{N} \sum_{i=1}^N \tau_{d23i}, \text{ i.e., each parameter is an average of parameters of } N \text{ number of firms/consumers of funds/capital.}$$

Producer of Labor

If producers of labor have heterogeneous technologies and have bounded rationality, their present value gets modified to the following expression:

$$V(0) = \int_0^\infty [\gamma_i p_{3i}(t) F_{li}(K_l(t), L_l(t)) - p_{2i}(t) I_l(t) - p_{li}(t) L_l(t)] e^{-r_{li}t} dt \quad (63)$$

Due to imperfect information, $p_{li}(t)$, $p_{2i}(t)$, and $p_{3i}(t)$ are prices known to producer which are assumed to be positive functions of actual market prices. This implies if $p_l(t)$, $p_2(t)$, and $p_3(t)$ increase, $p_{li}(t)$, $p_{2i}(t)$, and $p_{3i}(t)$ also increase respectively. γ_i being fraction of market price, i.e., p_{3i} charged by producer of labor to worker/laborer; r_{li} being discount rate; $L_l(t)$ (labor) and $I_l(t)$ (level of investment in terms of capital/funds/money with same price of capital as in goods market) as *control variables* and $K_l(t)$ being *state variable*. p_{2i} is price of capital, and p_{li} is wage/price of labor (this is input labor to produce type of skills/labor to be used in goods market). Maximization problem is as under:

$$\underset{\{L_l(t), I_l(t)\}}{\text{Max}} V(0) = \int_0^{\infty} [\gamma_i p_{3i}(t) F_{li}(K_l(t), L_l(t)) - p_{2i}(t) I_l(t) - p_{li}(t) L_l(t)] e^{-r_{li}t} dt,$$

subject to following constraints

$\dot{K}_l(t) = I_l(t) - \delta_{li} K_l(t)$ (state equation, describing how the state variable changes with time),

$K_l(0) = K_{l0}$ (initial condition),

$K_l(t) \geq 0$ (non-negativity constraint on state variable),

$K_l(\infty)$ free (terminal condition).

Current-value Hamiltonian is as under:

$$\tilde{H} = \gamma_i p_{3i}(t) F_{li}(K_l(t), L_l(t)) - p_{2i}(t) I_l(t) - p_{li}(t) L_l(t) + \mu(t) [I_l(t) - \delta_{li} K_l(t)] \quad (64)$$

Maximizing conditions are as under:

(i) $L_l^*(t)$ and $I_l^*(t)$ maximize \tilde{H} for all t : $\frac{\partial \tilde{H}}{\partial L_l} = 0$ and $\frac{\partial \tilde{H}}{\partial I_l} = 0$,

(ii) $\dot{\mu} - r_{li}\mu = -\frac{\partial \tilde{H}}{\partial K_l}$,

(iii) $\dot{K}_l^* = \frac{\partial H}{\partial \mu}$ (this just gives back the state equation),

(iv) $\lim_{t \rightarrow \infty} \mu(t) K_l(t) e^{-r_{li}t} = 0$ (the transversality condition).

Conditions (i) and (ii) can be expressed as follows:

$$\frac{\partial \tilde{H}}{\partial L_l} = \gamma_i p_{3i}(t) F'_{li2}(K_l(t), L_l(t)) - p_{li}(t) = 0 \quad (65)$$

$$\frac{\partial \tilde{H}}{\partial I_l} = -p_{2i}(t) + \mu(t) = 0 \quad (66)$$

And

$$\dot{\mu} - r_{li}\mu = -\frac{\partial \tilde{H}}{\partial K_l} = -[\gamma_i p_{3i}(t) F'_{li1}(K_l(t), L_l(t)) - \delta_{li}\mu(t)] \quad (67)$$

Substituting values of $\dot{\mu}$ and μ from eq. (66) in (67) yields

$$\gamma_i p_{3i}(t) F'_{li1}(K_l(t), L_l(t)) - (r_l + \delta_l) p_{2i}(t) + \dot{p}_{2i}(t) = 0 \quad (68)$$

If $p_3(t)$ (price of labor) increases, producer of labor faces following inequalities at existing level of investment and labor:

$$\gamma_i p_{3i}(t) F'_{li2}(K_l(t), L_l(t)) - p_{li}(t) > 0,$$

$$\gamma_i p_{3i}(t) F'_{li1}(K_l(t), L_l(t)) - (r_l + \delta_l) p_{2i}(t) + \dot{p}_{2i}(t) > 0$$

This implies that producer of labor increases production as the market price of their output (labor skills) increases. If inventory of goods (x) increases, price of goods goes down, which decreases production of goods and hence demand of labor, which will reduce price of labor used in goods production leading to a lower supply of labor due to following inequality faced by producer of labor:

$$\gamma_i p_{3i}(t) F'_{li2}(K_l(t), L_l(t)) - p_{li}(t) < 0$$

Similarly, if inventory of capital goes up (*ceteris paribus*), producer of labor faces following inequality and increases production of labor:

$$\gamma_i p_{3i}(t) F'_{li1}(K_l(t), L_l(t)) - (r_l + \delta_l) p_{2i}(t) + \dot{p}_{2i}(t) > 0$$

Above discussion implies the following expression:

$$W_{s3i}(\text{Labor}) = -K_{s31i} M_1(t - \tau_{s31i}) + U_{s32i} M_2(t - \tau_{s32i}) - K_{s33i} \varepsilon_3(t - \tau_{s33i}),$$

which implies average production will increase as follows:

$$\begin{aligned} \frac{1}{N} \sum_{i=1}^N W_{s3i}(\text{Labor}) = & -\frac{1}{N} \sum_{i=1}^N K_{s31i} M_1(t - \frac{1}{N} \sum_{i=1}^N \tau_{s31i}) + \\ & \frac{1}{N} \sum_{i=1}^N U_{s32i} M_2(t - \frac{1}{N} \sum_{i=1}^N \tau_{s32i}) - \frac{1}{N} \sum_{i=1}^N K_{s33i} \varepsilon_3(t - \frac{1}{N} \sum_{i=1}^N \tau_{s33i}), \end{aligned}$$

Or

$$W_{s3}(\text{Labor}) = -K_{s31} M_1(t - \tau_{s31}) + K_{s32} f_{s32} \mu M_2(t - \tau_{s32}) - K_{s33} \varepsilon_3(t - \tau_{s33}) \quad (69)$$

Where

$$W_{s3} = \frac{1}{N} \sum_{i=1}^N W_{s3i}, \quad K_{s31} = \frac{1}{N} \sum_{i=1}^N K_{s31i}, \quad K_{s32} f_{s32} \mu = \frac{1}{N} \sum_{i=1}^N U_{s32i}, \quad \tau_{s31} = \frac{1}{N} \sum_{i=1}^N \tau_{s31i}, \quad \tau_{s32} = \frac{1}{N} \sum_{i=1}^N \tau_{s32i},$$

$$\tau_{s33} = \frac{1}{N} \sum_{i=1}^N \tau_{s33i}, \text{ i.e., each parameter is an average of parameters of } N \text{ number of producers.}$$

Firm/Consumer of Labor

With heterogeneous preferences and bounded rationality, from eq. (6), if price of labor goes up, *ith* firm faces following inequality:

$$\alpha_i p_{1i}(t) F'_{i2}(K(t), L(t)) - p_{3i}(t) < 0,$$

which implies after a wage increase, *ith* consumer of labor demands lower quantity of labor. If inventory of goods goes up, $p_{1i}(t)$ decreases, and from eq. (6) and (9), firm/producer of goods (x) faces following inequalities at existing level of investment and labor:

$$\alpha_i p_{1i}(t) F'_{i2}(K(t), L(t)) - p_{3i}(t) < 0, \quad \alpha_i p_{1i}(t) F'_{i1}(K(t), L(t)) - (r_i + \delta_i) p_{2i}(t) + \dot{p}_{2i}(t) < 0$$

This implies that after an increase in goods inventory, producer of goods demands a lower quantity of labor. Similarly, if inventory of money goes up, capital becomes cheaper, and demand of labor (which compliments capital) by firm goes up due to following inequality faced by firm

$$\alpha_i p_{1i}(t) F'_{i1}(K(t), L(t)) - (r_i + \delta_i) p_{2i}(t) + \dot{p}_{2i}(t) > 0$$

Above discussion implies the following expression:

$$W_{d3i}(\text{Labor}) = -K_{d31i} M_1(t - \tau_{d31i}) + U_{d32i} M_2(t - \tau_{d32i}) - K_{d33i} P_3,$$

which implies average demand will change as follows:

$$\begin{aligned} \frac{1}{N} \sum_{i=1}^N W_{d3i}(\text{Labor}) = & -\frac{1}{N} \sum_{i=1}^N K_{d31i} M_1(t - \frac{1}{N} \sum_{i=1}^N \tau_{d31i}) + \\ & \frac{1}{N} \sum_{i=1}^N U_{d32i} M_2(t - \frac{1}{N} \sum_{i=1}^N \tau_{d32i}) - \frac{1}{N} \sum_{i=1}^N K_{d33i} P_3, \end{aligned}$$

Or

$$W_{d3}(\text{Labor}) = -K_{d31}M_1(t - \tau_{d31}) + K_{d32}f_{d32}\kappa M_2(t - \tau_{d32}) - K_{d33}P_3 \quad (70)$$

Where

$$W_{d3} = \frac{1}{N} \sum_{i=1}^N W_{d3i}, \quad K_{d31} = \frac{1}{N} \sum_{i=1}^N K_{d31i}, \quad K_{d32}f_{d32}\kappa = \frac{1}{N} \sum_{i=1}^N U_{d32i}, \quad K_{d33} = \frac{1}{N} \sum_{i=1}^N K_{d33i}, \quad \tau_{d31} = \frac{1}{N} \sum_{i=1}^N \tau_{d31i},$$

$$\tau_{d32} = \frac{1}{N} \sum_{i=1}^N \tau_{d32i}, \text{ i.e., each parameter is an average of parameters of } N \text{ number of firms/consumers of labor.}$$