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Research Article

Risk-dominant Equilibrium in Chicken and Stag-hunt Games with Different Dilemma Strengths

Ahmed S Elgazzar[1*]

[1]Mathematics Department, Faculty of Science, Arish University, Arish, Egypt

[*]**Correspondence to: Ahmed S Elgazzar, PhD, Professor,** Mathematics Department, Faculty of Science, Arish University, North Sinai, Arish 45516, Egypt; Email: aselgazzar@aru.edu.eg

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Abstract

Objective: This study investigates how the strength of risk-averting and gamble-intending dilemmas can affect the risk-dominant equilibrium in chicken and stag-hunt games. The focus is on deriving conditions under which the risk-dominant equilibrium can maximize group benefit.

Methods: Basic concepts of game theory were used to determine the Nash equilibria of each game.The Harsanyi-Selten theory was applied to determine the risk-dominant equilibrium. By manipulating the parameters related to risk-averting and gamble-intending dilemmas, different game scenarios are simulated to observe the resulting equilibria.

Results: The study found that for a given strength of the risk-averting dilemma, the risk-dominant equilibrium in the chicken game continuously shifts towards mutual cooperation as the strength of the gamble-intending dilemma decreases. In the SH game, the degree of cooperation of the risk-dominant equilibrium for a given strength of the gamble-intending dilemma decreases with increasing strength of the risk-averting dilemma and takes the discrete values: 1, 0.5 and 0. The results show the importance of understanding individual risk preferences to predict strategic decisions and equilibrium outcomes in these game settings.

Conclusion: The risk-dominant equilibrium in chicken and stag-hunt games depends on the relative strength of the risk-averting and gamble-intending dilemmas. By taking these factors into account, researchers and policy makers can better predict the likely outcomes of strategic interactions in scenarios with conflicting interests and different risk tolerances. The risk-dominant equilibrium can provide reasonable solutions to real-world conflict situations, such as the Iran-Iraq conflict over shared oil and gas resources.

Keywords: risk-dominant equilibrium, chicken game, stag-hunt game, multiple nash equilibria, Iran-Iraq conflict, risk-averting dilemma, gamble-intending dilemma

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1 INTRODUCTION

Game theory was originally formulated by von Neumann and Morgnestern[1] to mathematically model the behavior of rational agents in conflict situations. Since then, the applications of game theory have expanded to many other areas[2-10]. In a non-cooperative game, agents cannot communicate to reach agreements. Nash equilibrium (NE)[11] is a fundamental solution concept in game theory. It predicts a combination of actions that maximizes the agents' individual payoffs. If the agents would lose if they unilaterally changed their actions, then this combination of actions is called a strict NE. Many games and also many real-life situations of industrial organization and international trade have multiple NEs. Therefore, the choice of a unique equilibrium is important.

The pairwise nature is the basic pattern of interaction. Moreover, in many conflict situations there are only two possible actions. Therefore, the two-player, two-strategy (2×2) games are of particular importance. A rescaled formulation of 2×2 symmetric games is introduced by defining two universal dilemma strength parameters, namely the risk-averting dilemma and the gamble-intending dilemma[12-14]. Depending on the dilemma strength parameters, a 2×2 symmetric game is categorized into the following classes: trivial (T) (no dilemma), stag-hunt (SH)[15] (only risk-averting dilemma), chicken (CH)[16] (only gamble-intending dilemma), prisoner's dilemma (PD)[17] (both risk-averting and gamble-intending dilemmas). The T game is characterized by a unique optimal NE, while the PD game is characterized by a unique non-optimal NE. The SH game is characterized by two symmetric NEs, while the CH game is characterized by two asymmetric NEs. In both SH and CH games, players face the dilemma of choosing a unique NE in addition to the original dilemma. The study of these social dilemmas is of great importance for various economic aspects[18-21] and is the subject of intensive research[3,22-25].

Pareto-efficient equilibrium is an important concept in game theory. At a Pareto-efficient equilibrium, the outcome is such that no unilateral change can lead to an improvement in one player's payoff without reducing the payoff of the other player. In the CH game, a Pareto-efficient equilibrium usually results in both players taking risky actions, leading to a suboptimal outcome where both players can suffer a large loss. In a Pareto-efficient equilibrium in the SH game, both players are able to achieve a mutually beneficial outcome through cooperation, but both players are always worried that the opponent may deviate from the Pareto-efficient equilibrium. Pareto efficiency also has some fundamental limitations, for example, it does not take into account fairness in the distribution of payoffs between players.

Correlated equilibria[26,27] allow players to coordinate their strategies through external signals or recommendations in order to achieve mutual benefit. A correlated equilibrium can be defined as a probability distribution over joint actions that maximizes each player's expected payoff. In an experimental study[28], it was found that even when recommendations were only followed 70−75% of the time, they still had a significant impact on the distribution of outcomes. However, the use of correlated equilibria also has some disadvantages. First, correlated equilibria rely on external signals or correlations that may not always be available in real-world scenarios. Second, correlated equilibria can be risky if a player deviates from them.

Game theory has been used in several studies to analyze conflicts over shared resources[4], as it captures the strategic dynamics of decision-making. Game theory models allow for the consideration of cooperative and competitive behaviors and can provide insights into strategic decision-making and possible solutions. Madani[7], for example, has used CH, SH and PD games to analyze conflicts over water resources and has shown that these models can be useful in planning and policy making. Esmaeili et al.[9] and Araujo and Leoneti[10] have proposed a practical application of 2×2 games to analyze some conflicts in the oil and gas industry. They have shown that theoretical game models can improve decision-making processes in this area. The Iran-Iraq conflict over shared natural resources is an important example of such conflicts. It escalated into a military confrontation in the 1980s and lasted for eight years. This war led to considerable damage to the economy and infrastructure of both countries. Developing convincing solutions to the dilemmas of theoretical games can suggest solutions to many conflict situations in the real world.

Equilibrium selection in game theory is a crucial concept in analyzing strategic interactions between rational decision-makers. Focal point selection[29,30] is an approach to select a unique equilibrium. This approach is based on the idea that certain equilibria are more salient or natural for the players due to shared knowledge or expectations. Players may converge to a particular equilibrium simply because it stands out as a focal point. Focal point selection can identify a unique equilibrium even in the absence of clear dominance criteria. This method takes into account the common expectations of the players and the natural focal points, making it intuitive and easy to understand. However, it relies heavily on subjective assumptions and may not always lead to socially optimal outcomes.

Harsanyi and Selten[31] have introduced an algorithm for selecting a unique equilibrium in each 2×2 game with two strict pure equilibria. This theory gives absolute priority to the payoff-dominance criterion. If the payoff-dominance criterion is not satisfied, a risk-dominant equilibrium is selected based on a comparison of Nash products. Some researchers[32,33] have presented computational implementations of the Harsanyi-Selten algorithm. The Harsanyi-Selten approach and its computational implementations always fail in the case of games with more than two players. Recently, Cao

Table 1. Payoff Matrix of a General 2×2 Symmetric Game

	B: *C*	B: *D*
A: *C*	(*R*, *R*)	(*S*, *T*)
A: *D*	(*T*, *S*)	(*P*, *P*)

Table 2. Rescaled Payoff Matrix of a General 2×2 Symmetric Game With the Two Universal Scaling Dilemma Strength Parameters, D_g and D_r

	B: *C*	B: *D*
A: *C*	(*R*, *R*)	($P-D_r$, $R+D_g$)
A: *D*	($R+D_g$, $P-D_r$)	(*P*, *P*)

Table 3. Classes of the General 2×2 Symmetric Game According to the Dilemma Strength Parameters, D_g and D_r, and the Corresponding Pure NEs

Class	Dilemma Parameters	NE
T	$D_g<0$, $D_r<0$	(*C*, *C*)
SH	$D_g<0$, $D_r>0$	(*D*, *D*) (*C*, *C*)
CH	$D_g>0$, $D_r<0$	(*D*, *C*) (*C*, *D*)
PD	$D_g>0$, $D_r>0$	(*D*, *D*)

and Dang[34] presented an alternative approach for n-player games.

Zhang and Hofbauer[35] introduced quantal response equilibrium as a method for equilibrium selection in 2×2 coordination games. Their approach is based on the Nash product criterion and the limiting quantal response equilibrium. However, there are some limitations, such as the inconsistency of the priorities with the risk-dominant strategies in equilibrium selection processes. Moreover, small changes in these parameters can potentially alter the equilibrium outcomes selected by the quantal response equilibrium.

In non-cooperative games, one player does not know how the other player will play. Players would then choose the safer equilibrium rather than the more profitable one. Some studies[36,37] have shown that the risk-dominant equilibria are more common in evolutionary processes. Lee et al.[38] have shown that selection of the risk-dominant NE is robust under state-dependent mutation in local interaction games. The risk-dominant equilibrium is also supported by experimental results[39,40]. The risk-dominant equilibrium has many other advantages. It is invariant to player and strategy re-naming and to changes in payoffs that preserve the structure of the best-reply correspondence. Despite the intensive theoretical[41-44] and experimental[45,46] research on the risk-dominant equilibrium, there are no studies on the effects of dilemma strength parameters.

Our aim is to study the risk-dominant equilibrium in both SH and CH games. We consider a general formulation of 2×2 symmetric games with variable risk-averting and gamble-intending dilemmas. We analyze the efects of both risk-averting and gamble-intending dilemmas with respect to the risk-dominant equilibrium. We derive conditions under which the risk-dominant equilibrium solves both the SH and CH dilemmas. Finally, we apply our findings to two models of the Iran-Iraq conflict over shared oil and gas resources[9].

The remainder of this article is organized as follows. Section 2 defines a general 2×2 symmetric game with two dilemma strength parameters. The Harsanyi-Selten theory is briefly explained in Section 3. In Section 4, we study the risk-dominant equilibrium in both SH and CH games. The influence of risk-averting and gamble-intending dilemmas is emphasized. Section 5 is devoted to the application of the risk-dominant equilibrium to models of the Iran-Iraq conflict over shared oil and gas resources. Some conclusions are given in Section 6.

2 GENERAL 2×2 SYMMETRIC GAME

A general 2×2 symmetric game consists of two players A and B, and two pure actions: either to cooperate (*C*) or to defect (*D*). Let the payoffs for the reward, sucker, temptation and punishment be *R*, *S*, *T* and *P*, respectively. The payoff matrix is shown in Table 1. Depending on the order of the values of the payoff elements, *R*, *S*, *T* and *P*, different social dilemmas arise.

Tanimito et al.[12-14] have introduced two universal scaling parameters for the strength of the dilemma: $D_r=P-S$ for the risk-averting dilemma and $D_g=T-R$ for the gamble-intending dilemma. The rescaled payoff matrix is shown in Table 2. A positive D_r encourages a player to avoid losing by defecting when the opponent defects. A positive D_g encourages a player to increase profit by defecting when the opponent cooperates. Depending on the values of D_g and D_r, general 2×2 symmetric game is divided into four classes: T, SH[15], CH[16], and PD[17]. The four classes and the corresponding pure NEs are listed in Table 3.

It is important to identify the conditions that promote cooperative behavior in SH, CH and PD games. As can be seen in Table 3, players in the CH and SH classes face the additional dilemma of choosing a unique NE from the two possible ones. In a mixed-strategy game, the expected payoff functions are given in Equations (1) and (2).

$$\$_{\mathrm{A}}(p,q)=q[R+(1-p)D_g]+(1-q)(P-pD_r) \quad (1)$$

and

$$\$_{\mathrm{B}}(p,q)=p[R+(1-q)D_g]+(1-p)(P-qD_r) \quad (2)$$

where p (q) is the degree of cooperation of the strategy of player A (B). In equilibrium $\frac{\partial \$_{\mathrm{A}}}{\partial p}=\frac{\partial \$_{\mathrm{B}}}{\partial q}=0$. The group benefit is $\$_{\mathrm{G}}(p,q)=\$_{\mathrm{A}}(p,q)+\$_{\mathrm{B}}(p,q)$.

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Table 4. Payoff Matrix of a 2×2 Game with Two Strict NEs: (U_A, U_B) and (V_A, V_B)

	B: U_B	B: V_B
A: U_A	(a_{11}, b_{11})	(a_{12}, b_{12})
A: V_A	(a_{21}, b_{21})	(a_{22}, b_{22})

3 THE HARASANYI-SELTEN THEORY

To illustrate the Harsanyi-Selten theory[31], consider a 2×2 game with two strict NEs as described in Table 4. The loss of player A when deviating from the strict NEs, (U_A, U_B) and (V_A, V_B), while player B does not deviate, are a_{11}-a_{21} and a_{22}-a_{12}, respectively. The corresponding loss of player B are b_{11}-b_{12} and b_{22}-b_{21}. Depending on the best-reply structure of the game, there is a third mixed-strategy equilibrium point, (p^*, q^*), such that:

$$p^* = \frac{b_{22}-b_{21}}{(b_{11}-b_{12})+(b_{22}-b_{21})},\ q^* = \frac{a_{22}-a_{12}}{(a_{11}-a_{21})+(a_{22}-a_{12})} \quad (3)$$

Harsanyi and Selten[31] have introduced the following criteria:

(1) The payoff-dominance criterion: The NE that gives both players the highest payoffs should be selected.

(2) The risk-dominance criterion: If the payoff-dominance criterion is not satisfied, then the risk-dominant equilibrium should be chosen. The risk-dominant equilibrium is:

$$\begin{aligned}&(U_A, U_B), \text{if } (a_{11}-a_{21})(b_{11}-b_{12}) > (a_{22}-a_{12})(b_{22}-b_{21})\\ &(V_A, V_B), \text{if } (a_{11}-a_{21})(b_{11}-b_{12}) < (a_{22}-a_{12})(b_{22}-b_{21}) \quad (4)\\ &(p^*, q^*), \text{if } (a_{11}-a_{21})(b_{11}-b_{12}) = (a_{22}-a_{12})(b_{22}-b_{21})\end{aligned}$$

If the strict NEs lie on the other diagonal of the payoff matrix, i.e., (U_A, V_B) and (V_A, U_B), then the risk-dominant equilibrium is:

$$\begin{aligned}&(U_A, V_B), \text{if } (a_{12}-a_{22})(b_{12}-b_{11}) > (a_{21}-a_{11})(b_{21}-b_{22})\\ &(V_A, U_B), \text{if } (a_{12}-a_{22})(b_{12}-b_{11}) < (a_{21}-a_{11})(b_{21}-b_{22}) \quad (5)\\ &(\acute{p}^*, \acute{q}^*), \text{if } (a_{12}-a_{22})(b_{12}-b_{11}) = (a_{21}-a_{11})(b_{21}-b_{22})\end{aligned}$$

where

$$\acute{p}^* = \frac{b_{21}-b_{22}}{(b_{12}-b_{11})+(b_{21}-b_{22})} \quad (6)$$
$$\acute{q}^* = \frac{a_{12}-a_{22}}{(a_{21}-a_{11})+(a_{12}-a_{22})}$$

The Harsanyi-Selten theory[31] gives absolute priority to the criterion of payoff-dominance. However, we propose to give absolute priority to the risk-dominance criterion for the following reasons.

(1) Many studies[36-40] support the risk-dominant equilibrium.

(2) In non-cooperative games, agreements cannot be reached.

(3) It leads to risk-averse decisions. In real conflicts, averting risks often takes precedence over achieving benefits.

Therefore, players are more likely to choose the risk-dominant equilibrium. On the other hand, there is the limitation that the risk-dominant equilibrium may overlook the potential for higher payoffs. In the next section, we examine the risk-dominant equilibrium in both CH and SH games. We also establish the conditions under which the risk-dominant equilibrium becomes Pareto-efficient.

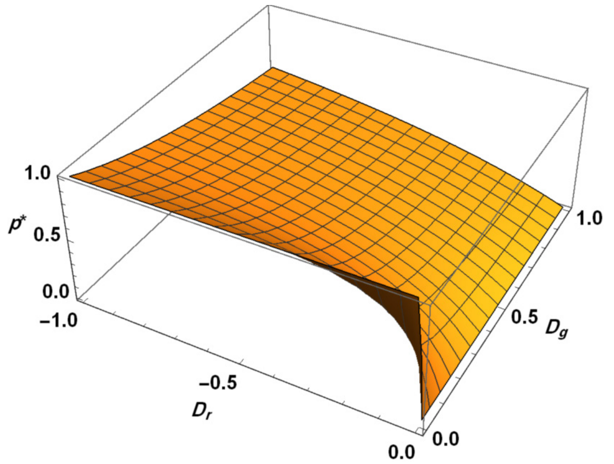

Figure 1. Illustration of the degree of cooperation of the risk-dominant equilibrium p^* of a CH game as a function of the dilemma strength parameters D_r and D_g. It is clear that p^* increases as $|D_r|$ increases and decreases as D_g increases. Moreover, p^* approaches 1 (0) when $|D_r| \gg D_g$ ($|D_r| \ll D_g$).

4 RISK-DOMINANT EQUILIBRIUM

Let us now apply the risk-dominant equilibrium to the two classes CH and SH and examine the effects of D_r and D_g.

4.1 The CH Class

In the CH game, D_g>0 and D_r<0, then we only have the gamble-intending dilemma. As can be seen in Table 3, players are faced with the dilemma of choosing one of the two unfair NEs: (D, C) and (C, D). In both equilibria, the D-player's payoff is greater than the C-player's payoff, then the D strategy would attract both players. In this case, each player receives the worst payoff, P. To determine the risk-dominant equilibrium, the Nash products are calculated as follows:

$$(a_{12}-a_{22})(b_{12}-b_{11}) = (a_{21}-a_{11})(b_{21}-b_{22}) = -D_rD_g \quad (7)$$

Therefore, the risk-dominant equilibrium of the CH class is the mixed-strategy equilibrium $(\frac{-D_r}{-D_r+D_g}, \frac{-D_r}{-D_r+D_g})$. This equilibrium point corresponds to simultaneous cooperation with degree $p^*=q^*=\frac{-D_r}{-D_r+D_g}$ and defection with degree $\frac{D_g}{-D_r+D_g}$. Such equilibrium points have attracted much interest in modeling decision-making during the COVID-19 pandemic[5,6]. The degree of cooperation, p^*, depends only on D_r and D_g. An illustration can be found in Figure 1. For a given D_g, p^* increases with increasing $|D_r|$. This increase is greater for lower values of D_g and vice versa. In the limit $|D_r| \gg D_g$, the gamble-intending dilemma can be neglected. Consequently, the risk-dominant equilib- rium approaches (C, C), and the expected payoffs approach the optimal payoffs (R, R), similar to the T game. In this limit, the risk-dominant equilibrium solves the CH dilemma. On the other hand, the degree of defection, 1-p^*,

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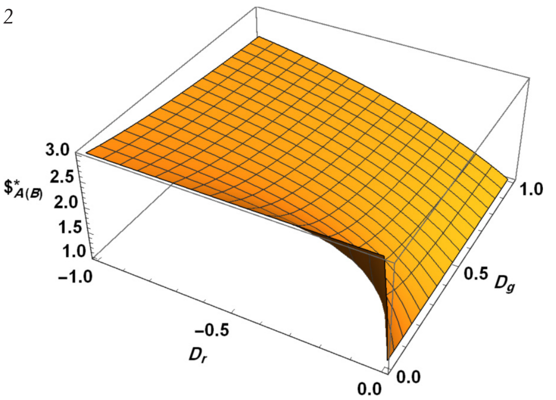

Figure 2. The expected payoff functions of the risk-dominant equilibrium of the CH game, $\$^*_{\mathrm{A(B)}}$. The influence of the dilemma strength parameters D_r and D_g is examined at constant pay-off elements, R=3 and P=1. Like p^*, $\$^*_{\mathrm{A(B)}}$ increases with increasing $|D_r|$ and decreases with increasing D_g.

increases with increasing D_g, while $|D_r|$ remains constant. This increase is greater at lower values of $|D_r|$, and vice versa. If $|D_r| \ll D_g$, the risk-dominant equilibrium goes to (D, D) with the worst expected payoffs (P, P) similar to the PD game. If $|D_r|=D_g$, the risk-dominant equilibrium is $(\frac{1}{2},\frac{1}{2})$. In this case, players choose their strategies randomly and each player's expected payoff is the average of the payoff matrix. This behavior is consistent with the behavior of players in some limits of evolutionary[25] and quantum[47-50] games.

Using Equations (1) and (2), the expected payoff functions of the risk-dominant equilibrium are given in Equation (8).

$$\$^*_{\mathrm{A(B)}} = \frac{1}{-D_r + D_g}(-D_rR + D_gP - D_rD_g)\ (8)$$

In Figure 2, we set arbitrary values for the payoff elements, R=3 and P=1 and investigate the influence of D_r and D_g. For a constant $|D_r|$, an increase in D_g leads to a decrease in $\$^*_{\mathrm{A(B)}}$, as p^* decreases. For a given D_g, an increase in $|D_r|$ leads to an increase in $\$^*_{\mathrm{A(B)}}$ as p^* increases. The group benefit due to the risk-dominant equilibrium is $\$_G(p^*,q^*) = 2\$^*_{\mathrm{A(B)}}$, while the group benefit due to (D, C) or (C, D) is $\$_G(D, C)=\$_G(C, D)=R+D_g+P-D_r$. It is clear that $\$_G(p^*, q^*)>\$_G(D, C)$ if $|D_r|>D_g$ and $R-P>\frac{D_r^2+D_g^2}{-D_r-D_g}$. In this case, the risk-dominant equilibrium improves the group benefit of the CH game.

As a measure of the risk associated with each equilibrium, we calculate the maximum loss incurred if one player chooses the equilibrium strategy while the other player deviates from it. For the equilibrium point (D, C), if player A decides to defect while player B deviates from cooperation, the expected payoff functions become:

$$\$_{\mathrm{A}}(D,q) = (R - P + D_g)q + P\ (9)$$

and

$$\$_{\mathrm{B}}(D,q) = -D_rq + P\ (10)$$

$\$_{\mathrm{A}}(D, q)$ is an increasing function of q. Its minimum value is P when player B defects. The maximum value of $\$_{\mathrm{A}}(D, q)$ is $R+D_g$ that is reached if player B cooperates. The maximum loss of player A, $\triangle_{max}(D, C)$, due to player B's deviation from cooperation is:

$$\triangle_{max}(D,C) = \$_{\mathrm{A}}(D,C) - \$_{\mathrm{A}}(D,q) = R + D_g - P\ (11)$$

For the risk-dominant equilibrium (p^*, q^*), if player A chooses the risk-dominant equilibrium strategy while player B deviates from it, the expected payoff functions become:

$$\$_{\mathrm{A}}(p^*,q) = (R - P + D_g + D_r)q + P + \frac{D_r^2}{-D_r + D_g}\ (12)$$

and

$$\$_{\mathrm{B}}(p^*,q) = \frac{1}{-D_r + D_g}(-D_rR + D_gP - D_rD_g)\ (13)$$

From the characteristics of the CH game, it follows that $\$_{\mathrm{A}}(p^*, q)$ is an increasing function of q. Its minimum value is $P+\frac{D_r^2}{-Dr+Dg}$, which is reached when player B defects. If player B cooperates, the maximum value of $\$_{\mathrm{A}}(p^*, q)$, $R+\frac{D_g^2}{-Dr+Dg}$ is reached. Player B's expected payoff, $\$_{\mathrm{B}}(p^*, q)$, is independent of his/her choice, q. It equals the expected payoff of the risk-dominant equilibrium. The maximum loss of player A, $\triangle_{max}(p^*, q^*)$, due to the deviation of player B from the risk-dominant strategy is:

$$\triangle_{max}(p^*,q^*) = \frac{1}{-D_r + D_g}[-D_r^2 - (R - P + D_g)D_r]\ (14)$$

At constant D_g, $\triangle_{max}(p^*, q^*)$ first increases when Dr decreases, then decreases and goes to zero when D_r goes to P-R-D_g. This happens when T-S is too small. At constant D_r, $\triangle_{max}(p^*, q^*)$ decreases when D_g increases until it stabilizes at the value -D_r when D_g is very large. The risk associated with (p^*, q^*) can be neglected if $D_g \gg |D_r|$. A comparison of Equations (11) and (14) clearly shows that $\triangle_{max}(p^*, q^*)<\triangle_{\mathrm{max}}(D, C)$. This ensures that the risk associated with (p^*, q^*) is lower than that associated with (D, C). Due to the symmetry of the game, the equilibrium (C, D) is also similar.

Let us take a numerical example. Let R=5, P=1, D_r=-1 and D_g=0.5. The risk-dominant equilibrium is the mixed-strategy equilibrium$(\frac{2}{3},\frac{2}{3})$. Equation (8) shows that the expected payoff for each player is 4. The group benefit is 8, which is greater than the group benefit resulting from either (D, C) or (C, D), namely 7.5. If player A chooses the risk-dominant equilibrium strategy and player B deviates from it, then

$$\$_{\mathrm{A}}(p^*,q) = \frac{7q}{2} + \frac{5}{3}\ (15)$$

and

$$\$_{\mathrm{B}}(p^*,q) = 4\ (16)$$

Then $\$_{\mathrm{A}}(p^*, q)$ is an increasing function in the range from $\frac{5}{3}$ to $\frac{31}{6}$, while $\$_{\mathrm{B}}(p^*, q)$ equals 4 (the expected payoff of the risk-dominant equilibrium). For each of the equilibria, (p^*, q^*) and (D, C), we calculate the maximum loss of player A due to the deviation of player B.

$$\triangle_{max}(p^*,q^*) = \frac{7}{3}\ (17)$$

$$\triangle_{max}(D,C) = \frac{9}{2}\ (18)$$

Table 5. Different Cases of the Risk-dominant Equilibrium of the SH Class and the Corresponding Expected Payoffs

Condition	Equilibrium	Expected Payoffs
$\lvert D_g \rvert > D_r$	(C, C)	(R, R)
$\lvert D_g \rvert = D_r$	$(\frac{1}{2}, \frac{1}{2})$	(π, π)
$\lvert D_g \rvert < D_r$	(D, D)	(P, P)

Notes: $\pi = \frac{1}{2}(R+P+D_g)$.

Table 6. Payoff Matrix of a Model for the Iran-Iraq Conflict Over the Fakka Oil Field[9]

	Iraq: Chicken	Iraq: Dare
Iran: chicken	(3, 3)	(2, 4)
Iran: dare	(4, 2)	(1, 1)

Notes: It is a CH Game with D_r=-1 and D_g=1.

Then the risk associated with (p^*, q^*) is less than the risk associated with (D, C).

4.2 The SH Class

In the SH game, D_g<0 and D_r>0, which represents only a risk-averting dilemma. This game has two pure NEs: (C, C) and (D, D). The equilibrium (C, C) guarantees the optimal payoff for each player. However, each player is very worried that the opponent might switch to the D strategy. On the other hand, the payoffs of the other equilibrium, (D, D), are the worst for both players. Now we determine the risk-dominant equilibrium. The Nash products become:

$$\begin{aligned}(a_{11}-a_{21})(b_{11}-b_{12}) &= (-D_g)^2 \\ (a_{22}-a_{12})(b_{22}-b_{21}) &= D_r^2\end{aligned} \quad (19)$$

Therefore, the risk-dominant equilibrium of the SH class is (C, C) if $|D_g|>D_r$, (D, D) if $|D_g|<D_r$ or $(\frac{1}{2}, \frac{1}{2})$ if $|D_g|=D_r$. The different cases of the risk-dominant equilibrium and the corresponding expected payoffs are summarized in Table 5.

If the loss a player would suffer by defecting if the opponent cooperates is greater than the loss a player can avoid by defecting if the opponent defects ($|D_g|>D_r$), then the risk-dominant equilibrium is (C, C). In this case, the risk-dominant equilibrium solves the SH dilemma. If player A chooses to cooperate while player B deviates from cooperation, the expected payoff functions become:

$$\$_A(C,q) = (R-P+D_r)q + P - D_r \quad (20)$$

and

$$\$_B(C,q) = -D_g q + R + D_g \quad (21)$$

Both $\$_A(C, q)$ and $\$_B(C, q)$ are increasing functions of q, such that $P-D_r \le \$_A(C, q) \le R$, and $R+D_g \le \$_B(C, q) \le R$. The maximum loss of player A, $\triangle_{max}(C, C)$, resulting from the deviation of player B from cooperation is:

$$\triangle_{max}(C,C) = R - P + D_r \quad (22)$$

Conversely, if the loss from defecting when the opponent cooperates is less than the gain from defecting when the opponent defects ($|D_g|<D_r$), the risk-dominant equilibrium is (D, D). If player A decides to defect while player B deviates, the expected payoff functions become:

$$\$_A(D,q) = (R-P+D_g)q + P \quad (23)$$

and

$$\$_B(D,q) = -D_r q + P \quad (24)$$

In this case, $\$_A(D, q)$ is an increasing function of q in the range from P to $R+D_g$, while $\$_B(D, q)$ is decreasing in the range from P to $P-D_r$.

If the loss from defecting when the opponent cooperates equals the gain from defecting when the opponent defects ($|D_g|=D_r$), the mixed-strategy $(\frac{1}{2}, \frac{1}{2})$ is the risk-dominant equilibrium. If player A chooses the risk-dominant equilibrium strategy while player B deviates from it, the expected payoff functions become:

$$\$_A(\frac{1}{2},q) = (R-P)q + P + \frac{D_r}{2} \quad (25)$$

and

$$\$_B(\frac{1}{2},q) = \frac{1}{2}(R+P+D_g) \quad (26)$$

Here $\$_A(\frac{1}{2}, q)$ is increasing from $P+\frac{D_r}{2}$ to $R+\frac{D_r}{2}$, while $\$_B(\frac{1}{2}, q)$ equals the expected payoff of the risk-dominant equilibrium, independent of q. The maximum loss of player A, $\triangle_{max}(\frac{1}{2}, \frac{1}{2})$, which results from the deviation of player B from the risk-dominant equilibrium, is:

$$\triangle_{max}(\frac{1}{2},\frac{1}{2}) = \frac{1}{2}(R-P) - D_r \quad (27)$$

It is clear that $\triangle_{max}(\frac{1}{2}, \frac{1}{2}) < \triangle_{max}(C, C)$.

5 APPLICATIONS TO THE IRAN-IRAQ CONFLICT

Here are two examples of the long-standing conflict between Iran and Iraq over their shared resources. The first is the Iran-Iraq conflict over the Fakka oil field (located on the Iran-Iraq border). This conflict is modeled by the CH game[9]. Both countries have an interest in exploiting the oil deposits in this area, leading to a risky situation in which neither country wants to back down for fear of forgoing potential benefits. The historical context of border disputes and previous conflicts between Iran and Iraq add to the tension of the situation. Failure to find a solution could lead to a disastrous outcome and escalate into a larger conflict. This mirrors the dynamics of a CH game where there is a risk of escalation. The decision-making process of both Iran and Iraq in this conflict resembled the strategic decisions in a CH game where each country has two options: Either dare to exploit the field or chicken out and leave it unexploited. Depending on the parametric formulation[9], the payoff matrix is given in Table 6. Modeling the Iran-Iraq conflict over the Fakka oil field as a CH game illustrates the strategic complexity and risk involved in such high-stakes situations. It also provides a framework for understanding the dynamics of the conflict and possible solutions.

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Table 7. Payoff Matrix of a Model for the Iran-Iraq Conflict Over Common Oil and Gas Resources[9]

	Iraq: *LER*	Iraq: *MER*
Iran: *LER*	(3, 3)	(0, 2)
Iran: *MER*	(2, 0)	(1, 1)

Notes: It is a SH Game with D_r=1 and D_g=-1.

In this model, there are two strict asymmetric and unfair NEs: (chicken out, dare) and (dare, chicken out). In equilibrium, only one country would benefit from the field. This would naturally encourage each country to dare, leading to a crash (war) in which each country receives the worst payoff, 1. In this model, D_r=-1 and D_g=1. The risk-dominant equilibrium is then the mixed-strategy $(\frac{1}{2},\frac{1}{2})$, which yields a payoff of $\frac{5}{2}$ for each country. This equilibrium point is a reasonable solution to this conflict and prevents fighting between the two countries.

The second application is to model the Iran-Iraq conflict over common oil and gas resources as an SH game[9]. The possible actions are either a low extraction rate (*LER*) or a maximum extraction rate (*MER*). Both countries can cooperate for the joint *LER* of shared resources to achieve optimal benefits, or both choose *MER*, leading to suboptimal outcomes. This corresponds to the concept of the SH game, where the optimal outcome is achieved when both players cooperate. However, unilateral actions or conflicting interests can lead to tensions and potential conflicts over the exploitation of resources. Based on Ref.[9], the payoff matrix is given in Table 7.

Modeling the Iran-Iraq conflict as an SH game highlights the potential benefits of cooperation and coordination in the management of shared oil and gas resources for mutual benefit. It also provides insights into the dynamics of cooperation, competition, strategic decision-making in resolving conflicts and maximizing the benefits of shared resources for both countries. This model has two strict NEs: (*LER, LER*) and (*MER*, *MER*). The equilibrium point (*LER*, *LER*) provides the optimal payoffs and guarantees the maximum long-term benefit for each country. However, there is a major concern that the opponent would turn into *MER*. The other equilibrium, (*MER*, *MER*), would completely destroy resources, and both countries receive the worst pay-offs. In this model, D_r=1 and D_g=-1. The risk- dominant equilibrium is then the mixed-strategy $(\frac{1}{2},\frac{1}{2})$, in which each country receives a payoff of $\frac{3}{2}$. This equilibrium is a good solution for the Iran-Iraq conflict. It preserves the rights of each country and guarantees the maximum long-term benefit of the shared oil and gas resources.

The solutions proposed here are convincing, ensure a balance between cooperation and competition and promote fairness between the two countries. But real-world conflicts are not just about payoffs and equilibria. The decision-making process in the real world should also take into account political, economic, social and historical factors. Nevertheless, game-theoretic models make an important contribution to the decision-making process in real-world conflicts over shared natural resources by providing a framework for analyzing and predicting the behavior of the actors involved in the conflict. In addition, game theory models can be used to identify areas where cooperation and negotiation can lead to better outcomes for all actors involved, rather than relying solely on competitive approaches.

6 CONCLUSION

When choosing a unique equilibrium for a non-cooperative game, we suggest the superiority of the risk-dominance criterion over the payoff-dominance criterion. We have applied the risk-dominance criterion to both CH and SH games with variable dilemma strength parameters. In the CH game, the cooperation degree of the risk-dominant equilibrium decreases continuously from 1 to 0 as the strength of the gamble-intending dilemma increases. As a result, the corresponding expected individual payoff decreases continuously in the range from *R* to *P*. Mutual cooperation is expected to be a risk-dominant equilibrium in an extreme limit of the CH game ($S\text{-}P \gg T\text{-}R$). Under certain conditions, the risk-dominant equilibrium can maximize the group benefit of the CH game.

In the SH game, the degree of cooperation of the risk-dominant equilibrium decreases with increasing strength of the risk-averting dilemma and takes the discrete values: 1, 0.5 and 0. Consequently, the corresponding expected individual payoff takes the discrete values: *R*, the average of the payoff matrix and *P*. The risk-dominant equilibrium of the SH game coincides with the payoff-dominant equilibrium (mutual cooperation) when $|D_g|>D_r$. Consequently, the SH dilemma with $R\text{-}T>P\text{-}S$ is solved using the risk-dominant equilibrium. We recommend redefining the condition for the SH dilemma as $R>T>P>S$ and $P\text{-}S>R\text{-}T$.

When $|R\text{-}T|=|P\text{-}S|$, the risk-dominant equilibrium of both SH and CH games is simultaneous cooperation and defection with equal probabilities to obtain an individual payoff equals the average of the payoff matrix.

These findings are important for game theory and for the many applications in other disciplines. We have applied the risk-dominance criterion to models of the Iran-Iraq conflict over shared oil and gas resources. The risk-dominant equilibrium could provide a reasonable solution to this conflict.

In real conflicts over shared resources, the decision-making process is not as simple as in the models presented here. Many other factors have to be taken into account. However, game-theoretic models can provide a reasonable insight into the strategic behavior of decision-makers without requiring a lot of precise quantitative information.

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Conflicts of Interest
The author declared no conflict of interest.

Author Contribution
The author contributed to the manuscript and approved the final version.

Abbreviation List
C, Cooperate
CH, Chicken
COVID-19, Corona Virus Disease 2019
D, Defect
LER, Low extraction rate
MER, Maximum extraction rate
NE, Nash equilibrium
PD, Prisoner's dilemma
SH, Stag-hunt
T, Trivial

References

[1] von Neumann J, Morgenstern O. Theory of Games and Economic Behavior. Princeton University Press: New Jersey, USA, 1944.
[2] Colman AM, Game Theory and its Applications in the Social and Biological Sciences. Butterworth-Heinemann: Oxford, UK, 1995.
[3] Tanimoto J. Fundamentals of Evolutionary Game Theory and its Applications. Springer: Tokyo, Japan, 2015.[DOI]
[4] Zara S, Dinar A, Patrone F. Cooperative game theory and its application to natural, environmental, and water resource issues: 2. Application to natural and environmental resources. World Bank Publications: Washington, USA, 2006.[DOI]
[5] Elgazzar AS. Coopetition in quantum prisoner's dilemma with different dilemma strength and COVID-19. *Quantum Inf Process*, 2021; 20: 102.[DOI]
[6] Crick JM, Crick D. Coopetition and COVID-19: Collaborative business-to-business marketing strategies in a pandemic crisis. *Ind Market Manag*, 2020; 88: 206-213.[DOI]
[7] Madani K. Game theory and water resources. *J Hydrol*, 2010; 381: 225-238.[DOI]
[8] Friedman D. On economic applications of evolutionary game theory. *J Evol Econ*, 1998; 8: 15-43.[DOI]
[9] Esmaeili M, Bahrini A, Shayanrad S. Using game theory approach to interpret stable policies for Iran's oil and gas common resources conflicts with Iraq and Qatar. *J Ind Eng Int*, 2015; 11: 543-554.[DOI]
[10] Araujo FC, Leoneti AB. Game theory and 2×2 strategic games applied for modeling oil and gas industry decision-making problems. *Pesqui Oper*, 2018; 38: 479-497.[DOI]
[11] Nash J. Equilibrium points in n-person games. *Proc Natl Acad Sci*, 1950; 36: 48-49.[DOI]
[12] Tanimoto J, Sagara H. Relationship between dilemma occurrence and the existence of a weakly dominant strategy in a two-player symmetric game. *Biosystems*, 2007; 90: 105-114.[DOI]
[13] Wang Z, Kokubo S, Jusup M et al. Universal scaling for the dilemma strength in evolutionary games. *Phys Life Rev*, 2015; 14: 1-30.[DOI]
[14] Ito H, Tanimoto J. Scaling the phase-planes of social dilemma strengths shows game-class changes in the five rules governing the evolution of cooperation. *R Soc Open Sci*, 2018; 5: 181085.[DOI]
[15] Skyrms B. The Stag Hunt and the Evolution of Social Structure. Cambridge University Press: Cambridge, UK, 2003.[DOI]
[16] Rapoport A, Chammah AM. The game of chicken. *Am Behav Sci*, 1966; 10: 10-28.[DOI]
[17] Ahmed E, Hegazi AS, Elgazzar AS. On some variants of prisoner's dilemma dynamics. *Appl Math Comput*, 2005; 163: 163-168.[DOI]
[18] Battalio R, Samuelson L, Huyck J. Optimization Incentives and Coordination Failure in Laboratory Stag Hunt Games. *Econometrica*, 2001; 69: 749-764.[DOI]
[19] Bech M, Garratt R. The intraday liquidity management game. *J Econ Theory*, 2001; 109: 198-219.[DOI]
[20] Coates C, Florence R, Kral K. Financial Statement Audits, a Game of Chicken. *J Bus Ethics*, 2002; 41: 1-11.[DOI]
[21] Barnett R. Inflation, taxes, and the coordination of monetary and fiscal policy by use of a game of chicken. *Can J Econ*, 2001; 34: 82-99.[DOI]
[22] Samle S. The prisoner's dilemma and dynamical systems associated to non-cooperative games. *Econmetrica*, 1980; 48: 1617-1634.[DOI]
[23] Ahmed E., Hegazi AS, Elgazzar AS. On spatial asymmetric games. *Adv Complex Syst*, 2002; 5: 433-443.[DOI]
[24] Nowak MA. Five rules for the evolution of cooperation. *Science*, 2006; 314: 1560-1563.[DOI]
[25] Ahmed E, Hegazi AS, Elgazzar AS. Sato-Crutchfield formulation for some evolutionary games. *Int J Mod Phys C*, 2003; 14: 963-971.[DOI]
[26] Aumann R. Subjectivity and Correlation in Randomized Strategies. *J Math Econ*, 1974; 1: 67-96.[DOI]
[27] Aumann R. Correlated Equilibrium as an Expression of Bayesian Rationality. *Econometrica*, 1987; 55: 1-18.[DOI]
[28] Duffy J, Feltovich N. Correlated equilibria, good and bad: an experimental study. *Int Econ Rev*, 2010; 51: 701-721.[DOI]
[29] Schelling TC. The Strategy of Conflict. Harvard University Press: Cambridge, UK, 1960.
[30] Sugden R. A theory of focal points. *Econ J*, 1995; 105: 533-550.[DOI]
[31] Harsanyi J, Selten R. A General Theory of Equilibrium Selection in Games. MIT Press: Cambridge, UK, 1988.
[32] Van den Elzen AH, Talman AJJ. An algorithmic approach towards the tracing procedure of Harsanyi and Selten. Tilburg University: Tilburg, Netherlands, 1995.
[33] von Stengel B, van den Elzen A, Talman D. Computing normal form perfect equilibria for extensive two-person games. *Econometrica*, 2002; 70: 693-715.[DOI]
[34] Cao Y, Dang C. A variant of Harsanyi's tracing procedures to

select a perfect equilibrium in normal form games. *Games Econ Behav*, 2022; 134: 127-150.[DOI]
[35] Zhang B, Hofbauer J. Quantal response methods for equilibrium selection in 2×2 coordination games. *Games Econ Behav*, 2016; 97: 19-31.[DOI]
[36] Kandori M, Mailath GM, Rob R. Learning, mutation and long-run equilibrium in games. *Econometrica*, 1993; 61: 29-56.[DOI]
[37] Young P. The evolution of conventions. *Econometrica*, 1993; 61: 57-84.[DOI]
[38] Lee IH, Szeidl A, Valentinyi A. Contagion and state dependent mutations. *BE J Theor Econ*, 2003; 3: 1-29.[DOI]
[39] Cabrales A, García-Fontes W, Motta M. Risk dominance selects the leader: An experimental analysis. *Int J Ind Organ*, 2000; 18: 137-162.[DOI]
[40] Cabrales A, Nagel R, Armenter R. Equilibrium selection through incomplete information in coordination games: an experimental study. *Exp Econ*, 2002; 10: 221-234.[DOI]
[41] Zhang B, Hofbauer J. Equilibrium selection via replicator dynamics in 2×2 coordination games. *Int J Game Theory*, 2015; 44: 433-448.[DOI]
[42] Cui Z. Linking friction, social coordination and the speed of evolution. *Games Econom Behav*, 2023; 140: 410-430.[DOI]
[43] Kamijo Y. A note on the risk dominance of the Nash demand game. *Int J Game Theory*, 2023; 2023: 1-11.[DOI]
[44] Arigapudi S. Transitions between equilibria in Bilingual Games under Probit Choice. *J Math Econom*, 2024; 111: 102956.[DOI]
[45] Meub L, Proeger T, Bizer K et al. Strategic coordination in forecasting - An experimental study. *Finance Res Lett*, 2015; 13: 155-162.[DOI]
[46] Castillo M, Dianat A. Strategic uncertainty and equilibrium selection in stable matching mechanisms: experimental evidence. *Exp Econ*, 2021; 24: 1365-1389.[DOI]
[47] Özdemir Ş K, Shimamura J, Imoto N. Quantum advantage does not survive in the presence of a corrupt source: optimal strategies in simultaneous move games. *Phys Lett A*, 2004; 325: 104-111.[DOI]
[48] Chen LK, Ang H, Kiang D et al. Quantum prisoner dilemma under decoherence. *Phys Lett A*, 2003; 316: 317-323.[DOI]
[49] Elgazzar AS. Unique solution to the quantum prisoner's dilemma game. *J Phys Soc Jpn*, 2019; 88: 034801.[DOI]
[50] Elgazzar AS. Quantum prisoner's dilemma in a restricted one-parameter strategic space. *Appl Math Comput*, 2020; 370: 124927.[DOI]