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Research Article

Newton-GMRES Method for the Solution of Piezo-Viscous Line Contact EHL Problem with Daubechies Wavelet as a Preconditioner

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Abstract

Objective: In the present paper, numerical analysis of the effect of piezo-viscous on line contact elastohydrodynamic lubrication (EHL) problem was performed.

Methods: The mathematical formulation of governing equations viz. Reynolds and film thickness equations were solved using Newton-Generalized minimal residual (GMRES) method with Daubechies wavelet as preconditioner.

Results: The study pertained to a wide range of parameters involved in the governing equations with the Newton-GMRES scheme, which features lower complexity and overcomes the limitations of conventional numerical schemes. The result showed that the pressure spike was strongly influenced by the piezo-viscous effect and sliding ratio, and the minimum film thickness was accurately measured, suggesting the importance of piezo-viscous effects on the EHL problem.

Conclusion: The Newton-GMRES method with wavelet preconditioner is one of the most efficient numerical methods and an alternative scheme to other classical numerical methods for the solution of EHL problem.

Keywords: elastohydrodynamic lubrication, Krylov subspace, preconditioner matrix

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1 INTRODUCTION

Elastohydrodynamic lubrication (EHL) is a regime of lubrication characterized by high pressure in the contact region and elastic deformation of lubricated surfaces, which requires significant attention in non-conformal contact of the mechanical components such as cams, gears, tappets, and roller-bearing. The unavailability of the EHL theory in predicting the accurate realistic lubricant film thickness for non-conformal contacting geometries at high pressures in the contacted region is attributable to the alterations in calculating the elastic deformation and piezo-viscous effect of the increasing lubricant viscosity. The film thickness between contacting materials is essential to ensure the protection of mating surfaces, and the traction coefficient of lubricants is a measure of performance viz. wear, power loss, degradation of lubricant. Therefore, traction coefficient measurement is an important aspect in EHL applications.

Prior EHL studies were mainly performed based on the isothermal Newtonian fluid model. Grubin and Vinogradova[1] first analyzed EHL contact, incorporating both elastic deformation and viscosity-pressure effects. For Newtonian fluid model, Dowson and Higginson[2] and Hamrock and Dowson[3] acquired the numerical solution of EHL line and point contact problems. Ree and Eyring[4] proposed a non-Newtonian rheological model. Bair and Winer[5] incorporated limiting shear stress strength of lubricant and presented a nonlinear governing (constitutive) equation based on experiment conducted in laboratory. Gecim and Winer[6] depicted film thickness reduction up to 40% compared to Newtonian models. Evans and Johnson[7] described the increase in shear stress with increasing shear rate for the non-Newtonian model. Conry et al.[8] proposed a solution to the line contact problem for isothermal EHL non-Newtonian Re-Eyring fluids and reduced it to the limit of the Newtonian model. Khonsari and Hua[9] observed that the constitutive equations of non-Newtonian fluids with limiting shear stress exhibited simpler form and that the Reynolds-type equation is derived from perturbation schemes for other fluids. Using Bair-Winer's constitutive equation, Khonsari and Hua[10] presented a generalized Newtonian EHL model and demonstrated that the traction coefficients are consistent with experiment values.

Based on the physical concept, Doolittle[11] developed a free volume viscosity model in which the resistance to flow of a fluid depends on relative volume molecules present per unit free volume. Bair and Qureshi[12] introduced a Newtonian model for the shear thinning response and concluded that the power law form such as Carreau viscosity model is necessary for the description of traction and film thickness. Mongkolwongrojn et al.[13] used the Carreau viscosity model and discussed the characteristics of EHL line contact problems. Bair[14] confirmed the validity of the free volume model based on high pressure viscometer results. Jang et al.[15] presented an algorithm for non-Newtonian fluid, comprising the Carreau model for the EHL problem that achieves similar prediction with experiment values. Chapkov et al.[16] and Liu et al.[17] used modified Carreau viscosity model with the free volume model and found consistent experiment and predicted values of the film thickness. Kumar and Khonsari[18,19] analyzed the EHL properties for line contact with effect of shear thinning and viscous heat, and proposed correction factor for shear-thinning fluid in EHL point contact. Kumar et al.[20] revealed a significant variation in pressure and film thickness under the influence of polymeric additives. Newtonian quantitative EHL film thickness with linear piezovisosity was discussed by Kumar et al[21]. Kumar and Kalita[22] considered shear thinning and linear piezo-viscous fluids for simulations during sudden halt and impact loading to normal approach of transient EHL. Li et al.[23] studied the properties of EHL of a series of oil droplets when entrained along the center line of the contact zone. Hultquist et al.[24] used Tiat's equation and analyzed a finite line EHL contact and its influence on subsurface stress concentrations in the contacting body.

In most studies, Newton-Raphson method is applied for the numerical solution of EHL regime. The computational complexity of the method is o (n^3) as the grid size n increases with the increase of the sparsity of the Jacobian matrix, and the computation in the spatial domain a large amount of time. Accordingly, the multigrid method is used to overcome this difficulty because it consumes less computational time and computation complexity i.e. o (nlogn). In this method, the computational domain was divided into Gauss Seidel iteration method for high pressure zone (Poisuille term of governing equation dominate compared to wedge term) and Jacobi di-pole method for low pressure zone (wedge term dominant compared to Poisuille term).

1.1 Krylov Subspace Methods (KSMs)

The classical iterative methods or other exact methods of solving system of equations are invalid provided the coefficient matrices are non-singular with a large condition number. These equations are often found in engineering and real-world scientific studies approaches. Iterative methods with preconditioned KSMs are an efficient and robust technique to solve systems of equations. Previous studies[25,26] introduced the theoretical ideas for the construction of preconditioners, and the skillful implementation of preconditioner techniques has been investigated in various research. The basic building blocks of KSMs are Lanczos and Arnoldi algorithms for deriving the solution to systems of algebraic linear equations[27]. Householder and Bauer[27] extended the method with matrix characteristic polynomials and derived relations with various methods, such as Lanczos' method. Hestenes and Stiefel[28] proposed the conjugate gradient method a direct method for symmetric positive definite matrices, and Reid[29] proposed KSM as an iterative method. Using symmetric Lanczos method, Paige and Saunders[30] significantly contributed in development of other KSMs, namely the minimum residual method. Saad and Schult[31] extended the Arnoldi algorithm by introducing the full orthogonalization method and generalized minimal residual (GMRES). In general, KSMs consist of different generic algorithms that have been coarsely categorized into three main groups; Arnoldi, Hermitian Lanczos and non-Hermitian Lanczos[25].

In this paper, Daubechies D6 wavelet was used as a preconditioner for the solution of the EHL problem using Newton-GMRES method. Using the preconditioner, the condition number of the Jacobian matrix was reduced to near unity, and thus the solution was obtained using Krylov subspace with required residual tolerance. This method does not require inversion of the Jacobian matrix and partition of the domain. Chen[26] proposed a method of preconditioning the sparse dense matrix. Ford et al.[32] used the restarted Newton-GMRES method with Daubechies D4 discrete wavelet with permutation to determine the solution of isothermal EHL line problem, which suggested that preconditioned matrix with banded matrix the computational complexity could be further reduced by focusing on computational cost in time.

The paper focused on Newton-GMRES method with D6 wavelet as preconditioner for the solution of EHL line contact with piezoviscous fluid. Pressure and film thickness profiles were obtained by solving the governing equations such as Reynolds, film thickness, and load balancing equations, including the Carreau shear-thinning model and Doolittle free volume pressure-viscosity equation with Tait's equation of state. The obtained results were presented in graphical form, including pressure and film thickness profiles. The characteristics of pressure spike, minimum film thickness and central film thickness were compared with earlier numerical and experimental results.

2 METHODS

2.1 Mathematical Formulation

The present analysis includes the viscosity model proposed by Bair and Qureshi[33] describing the shear thinning response derived from the generalized Newtonian model, which necessitates the description of both traction and film thickness in terms of power-law form such as Carreau viscosity mode in dimensionless form as

$$\bar{\eta} = \frac{\eta}{\mu_0} = \bar{\mu}\left[1+\left(\frac{\pi U \bar{\mu}\bar{\dot{\gamma}}}{8WH\,\bar{G}_{cr}}\right)^2\right]^{\frac{(n-1)}{2}} \quad (1)$$

where = u/u_0 is the low shear viscosity, n is the power-law index, $\bar{\dot{\gamma}} = \partial\bar{u}/\partial Y$ is the shear-strain rate, and $\bar{G}_{cr} = G_{cr}/E'$ is the critical stress corresponding to the Newtonian limit of lubricant in dimensionless form.

The Reynolds equation for any generalized Newtonian constitutive equation for EHL line contact with compressible fluid in the dimensionless form[34] is

$$\frac{\partial}{\partial X}\left(\bar{\rho} H^3 \bar{F}_2 \frac{\partial P}{\partial X}\right) = K\frac{\partial}{\partial X}\left(\bar{\rho} H\right) + K\frac{S}{2}\frac{\partial}{\partial X}\left[\bar{\rho} H\left(1-2\frac{\bar{F}_1}{\bar{F}_0}\right)\right] \quad (2)$$

Where $K = U\left(\frac{\pi}{4W}\right)^2$ and $S = \frac{(u_2-u_1)}{u_0}$ is the slide-to-roll ratio. The effective viscosity integral functions in the above equation are given as follows

$$\bar{F}_0 = \int_0^1 \frac{1}{\bar{\eta}}dY,\ \bar{F}_1 = \int_0^1 \frac{Y}{\bar{\eta}}dY \text{ and } \bar{F}_2 = \int_0^1 \frac{Y}{\bar{\eta}}\left(Y-\frac{\bar{F}_1}{\bar{F}_2}\right)dY \quad (3)$$

The boundary conditions for the proposed problem in the dimensionless form are given by

$$P = 0 \quad \text{at} \quad X = X_{in} \quad (4)$$

$$P = \frac{\partial P}{\partial X} = 0 \quad \text{at} \quad X = X_{out} \quad (5)$$

The film thickness equation in dimensionless form is

$$H(X) = H_0 + \frac{X^2}{2} + \frac{-1}{2\pi}\int_{X_{in}}^{X_{out}} P(X)\ln(X-X')dX' \quad (6)$$

For compressible fluids, Tait's equation of state is used for the density-pressure relationship and is given by

$$\frac{\rho_0}{\rho} = \frac{V}{V_0} = 1-\frac{1}{1+K'_0}\ln\left[1+\frac{p}{K_0}\left(1+K'_0\right)\right] \quad (7)$$

Where p is the lubricant density at a pressure p and p_0 is the lubricant density at p=0. The constants K_0 and EMBED Equation (3) in the above equation may be determined experimentally. The free volume Doolittle's pressure viscosity equation is used and is given as

$$\mu = \mu_0 \exp\left(B\frac{V_\infty}{V_0}\left[\frac{1}{\frac{V}{V_0}-\frac{V_\infty}{V_0}} - \frac{1}{1-\frac{V_\infty}{V_0}}\right]\right) \quad (8)$$

Where u is the lubricant viscosity at a pressure and u_0 is the lubricant viscosity at p=0. The constant B, V_∞, V_0 are experimentally determined lubricant properties and V/V_0 is determined from Equation (7). The dimensionless load balance equation in dimensionless form is given as

$$\int_{X_{in}}^{X_{out}} P dX = \frac{\pi}{2}. \quad (9)$$

2.2 Discretization of Governing Equations

The finite difference approximations were used to discretize Equations(2), (6) and (9), with grid size N (the number of grid points). The interval [X_{in}, X_{out}]=[-4,1.5] is spatial domain of computation and the cavitation point is to be determined during the solution process by letting negative pressure value is equal to zero. The governing equations along with their boundary conditions are discretized as

$$\frac{\varepsilon_{i-1/2}\,P_{i-1} - \left(\varepsilon_{i-1/2}+\varepsilon_{i+1/2}\right)P_i + \varepsilon_{i+1/2},\ P_{i+1}}{\Delta X^2} = K\frac{\bar{\rho}_i\ H_i\omega_i - \bar{\rho}_{i-1}H_{i-1}\omega_{i-1}}{\Delta X} \quad (10)$$

where $\Delta X = X_i - X_{i-1}$, $\varepsilon_i = \bar{\rho}_i\left(\bar{F}_2\right)H_i^3$, $\omega_i = 1+\frac{S}{2}\left(1-2\frac{\bar{F}_1}{\bar{F}_o}\right)_i$

and $\varepsilon_{i\pm 1/2} = \dfrac{\varepsilon_i + \varepsilon_{i\pm 1}}{2}$.

$$P(X_{in}) = 0 \quad \text{and} \quad \frac{P(X_{out}) - P(X_{out-1})}{\Delta X} = 0. \quad (11)$$

The discretized form of the film thickness equation becomes

$$H_i = H_0 + \frac{X_i^2}{2} - \frac{1}{\pi}\sum_{j=1}^{N+1} D_{i,j} P_j \quad (12)$$

where

$$D_{i,j} = \left(i - j + \frac{1}{2}\right)\Delta X\left[\ln\left(\left|i - j + \frac{1}{2}\right|\Delta X\right) - 1\right]$$
$$-\left(i - j - \frac{1}{2}\right)\Delta X\left[\ln\left(\left|i - j - \frac{1}{2}\right|\Delta X\right) - 1\right],$$

for, i, j=1,2,3,…, N+1. The discretized form of load balance equation given as

$$\Delta X \sum_{i=1}^{N}\left(\frac{P_i + P_{i+1}}{2}\right) - \frac{\pi}{2} = 0. \quad (13)$$

2.3 Numerical Solution

2.3.1 Newton-GMRES Method

The linear system of algebraic equations,

$$Ax=b \quad (14)$$

Where A is a dense, non-symmetric nxn square matrix, x is an unknown column matrix and b is a known column matrix. Equation (14) is solved using a Krylov subspace iterative method namely the GMRES method. However, given the slow convergence of the GMRES method, matrix A is first preconditioned to obtain better efficiency. Equation (14) is rewritten in right preconditioned form as

$$AP^{-1}y=b, \text{ where } y=Px \quad (15)$$

The preconditioner P is constructed to satisfy the following criteria;

1. AP^{-1} *is a* faster convergence when the GMRES method is used to solve Equation (15).
2. $P^{-1}v$ is calculated for any vector v very easily.

To approximate matrix $\boldsymbol{A}$, the standard discrete wavelet transform (DWT) comprising D6 wavelet was first considered. Chen[26] proposed the permutation of discrete D4 and D6 Daubechies wavelet transformations. All the results are computed by using the permutation of discrete D6 Daubechies wavelet transform.

Let m be the order of compactly supported wavelets with m low pass filter coefficients $a_0, a_1, a_2, \ldots, a_{m-1}$ and $m/2$ vanishing movements. The coefficients $b_0, b_1, \ldots, b_{m-1}$ are derived from a_1 by the relation of the form $b_i=(-1)^i a_{m-1-1}$. Also let $n=2^L$ and r is an integer such that 2^r<m and, where L is the resolution level in the Krylov subspace $0<r<L$. Let us denote $s=s^{(L)}$ a column vector of A at the wavelet level L and is a given function. The wavelet transform is implemented by the well-known pyramidal algorithm. The standard pyramidal algorithm transforms the vector $s^{(L)}$ to $w=\left[\left(s^{(r)}\right)^{T}\left(f^{(r)}\right)^{T}\left(f^{(r+1)}\right)^{T}\ldots\left(f^{(L-1)}\right)^{T}\right]^{T}$ in a level by level manner, where and are of length and sum of all these length is 2^L.

At a general level and denotes the coefficients collections of scaling and wavelet functions respectively. In matrix form, is expressed as

$$w = P_{r+1}W_{r+1}P_{r+2}W_{r+2}\ldots P_{L-1}W_{L-1}P_L W_L = Ws^{(L)}$$

where $P_v = \begin{pmatrix} \overline{P}_v & \\ & J_v \end{pmatrix}_{n\times n}$

with $\overline{P}_v$ is a permutation matrix of size 2^v, i.e.

$$\overline{P}_v = I(1,3,\ldots,2^v-1,2,4,\ldots,2^v)$$

and $W_v = \begin{pmatrix} \overline{W}_v & \\ & J_v \end{pmatrix}_{n\times n}$

where J_v is an identity matrix of size k^v and $\overline{W}_v$ is an orthogonal(sparse) matrix of size $2^v=2^L-k_v=n-k_v$. In this case, $k_L=0$ and $K_v=K_{v+1}+2^v$ for $v=L-1,\ldots, r+1$.

The one-level transformation matrix $\overline{W}_v$ with Daubechies order wavelets with vanishing moments is given by

$$\overline{W} = \begin{pmatrix}
a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & & & & & & \\
b_0 & b_1 & b_2 & b_3 & b_4 & b_5 & & & & & & \\
& & a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & & & & \\
& & b_0 & b_1 & b_2 & b_3 & b_4 & b_5 & & & & \\
& & & & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & \\
& & & & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & \\
& & & & & & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
& & & & & & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
a_4 & a_5 & & & & & & & a_0 & a_1 & a_2 & a_3 \\
b_4 & b_5 & & & & & & & b_0 & b_1 & b_2 & b_3 \\
a_2 & a_3 & a_4 & a_5 & & & & & & & a_0 & a_1 \\
b_2 & b_3 & b_4 & b_5 & & & & & & & b_0 & b_1
\end{pmatrix}_{n\times n}$$

where the filtering coefficients $\{a_i, b_i\}$ are known to be a_0=0.33267, a_1=0.80689, a_2=0.45987, a_3=-0.13501, a_4=0.08544, a_5=0.03522 and $b_k=(-1)^k a_{m-1-k}$.

2.3.2 Discrete Wavelet Transformation with Permutation (DWTPer)

For a matrix A, DWTPer is written as

$$\hat{A} = \hat{W}A\hat{W}^{T}.$$

where $P = P_L^{\mathrm{T}} P_{L-1}^{\mathrm{T}} \ldots P_{r+2}^{\mathrm{T}} P_{r+1}^{\mathrm{T}}$ and $\widehat{W} = PW$ are permutation matrix. D6 DWT with permutation is applied on the Jacobian matrix and thresholded, i.e., elements below a certain value are set to zero. A simple preconditioner constructed for such a Jacobian matrix, and a banded matrix is obtained by setting elements of the matrix located in the selected diagonal entries to zero. Band matrices are stored efficiently, especially with a good preconditioner and can be manipulated at low cost.

When D6 wavelet transforms are applied to the sparse and dense matrix, the nonzero elements increase as compared to D4 wavelet transform. Thus, the accuracy of the convergence of iteration method may also increases. Previous work[26] showed that it was difficult to design an preconditioner using DWT efficiently under all circumstances. A modified DWT was used to overcome the difficulty, with a centralized feature and diagonal bands. This new DWT provides robust preconditioning suitable for EHL simulation.

3 RESULTS AND DISCUSSION

The numerical investigation of line contact EHL problem with the effect of piezo-viscous was analyzed in detail. The finite difference solution of governing Reynolds and film thickness equations were solved by using Newton-GMRES method. The pressure profiles and lubricant film thickness profiles were depicted in the calculation domain *-4<X<1.5*. In this problem, a preconditioner was used to reduce the condition number of the Jacobian matrix which is close to one and converges to acquire less central processing unit time as compared to other numerical methods.

Figure 1A represents the pressure and film thickness distributions of the Carreau type of free-volume model, which was obtained numerically with constant load W=2E-5 for various values of speed with sliding ratio S=0. Figure 1B presents the profiles of pressure and film thickness for sliding ratio S=1 with same load W=2E-5 and various values of speed. Doolittle model was used for pressure-viscosity relationship and a pressure spike was predicted at the outlet of the contact region. Simultaneously a constriction was observed in lubricant film thickness profiles. The height of the pressure spike increased with an increase in the speed and moved towards inlet from outlet of the contact zone. As the load increased, the peak of the pressure spike became sharper and moved towards the outlet zone. The height of the meatus increased in the profiles of film thickness for increase in the dimensionless speed, whereas a reduction of the film thickness was observed with increase in the loads. (Figure 1B-1D).

Increasing power index n=0.75 and material parameter 3KPa, the height of the pressure spike increases for both sliding ratios S=0 and S=1, whereas the minimum and central film thickness increased slightly compared to n=0.5 (Figure 2). For sliding ratio S=1, the pressure spike are almost compressed above S=0 (Figure 3A). The minimum

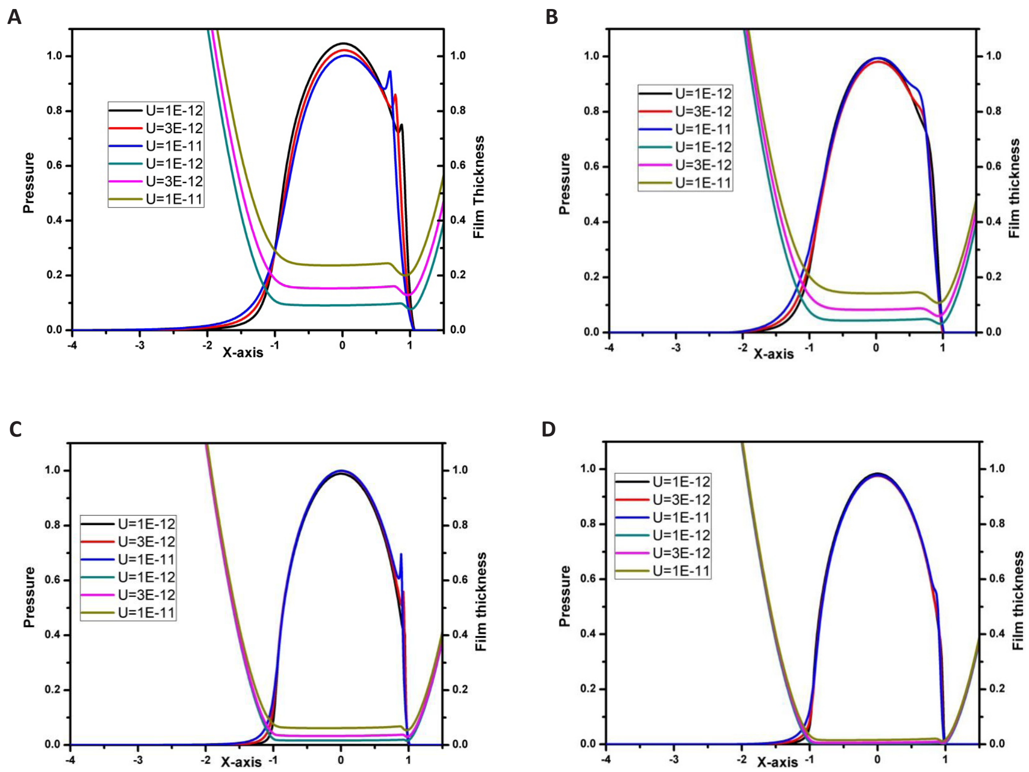

Figure 1. Pressure and film thickness profiles at W=2E-5, S=0 (A), W-2E-5, S=1 (B), W=1E-4, S=0 (C), and W-1E-4, S=1 (D) for various values of speed.

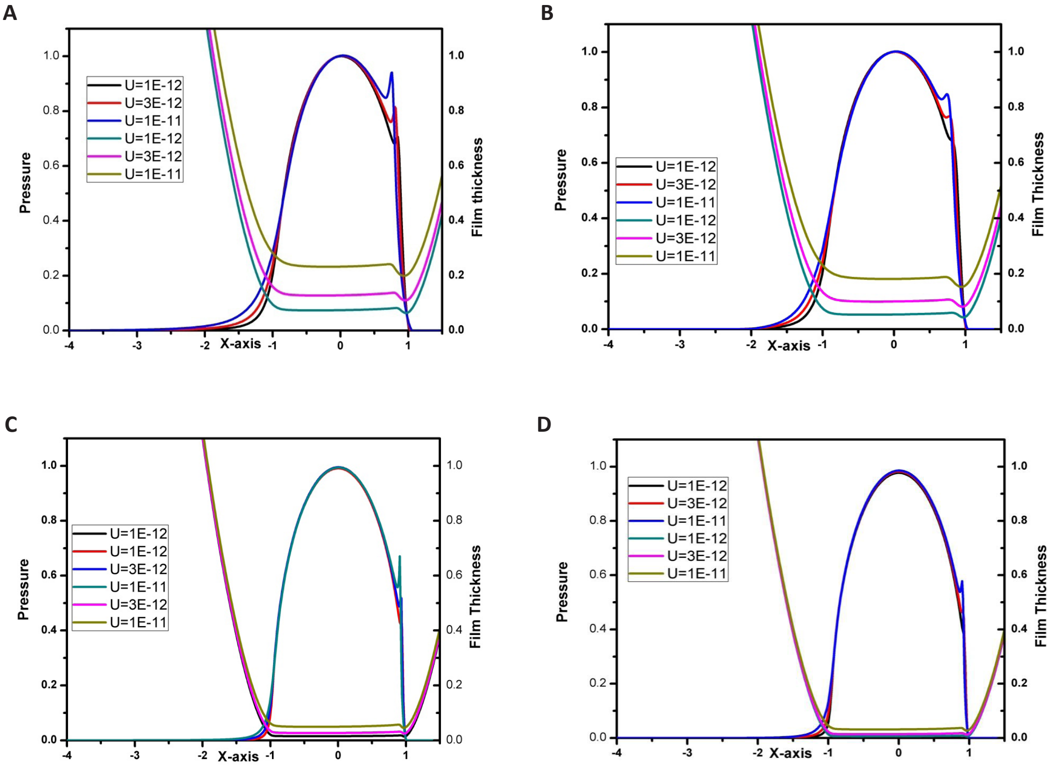

Figure 2. Pressure and film thickness profiles at G_{cr}=3Kpa, W=2E-5, S=0,n=0.75 (A), at G_{cr}=3Kpa, W=1E-4, S=0, n=0.75 (B), at G_{cr}=3Kpa, W=2E-5, S=1 ,n=0.75 (C), and at G_{cr}=3Kpa, W=1E-4, S=1, n=0.75 (D) for various values of speed.

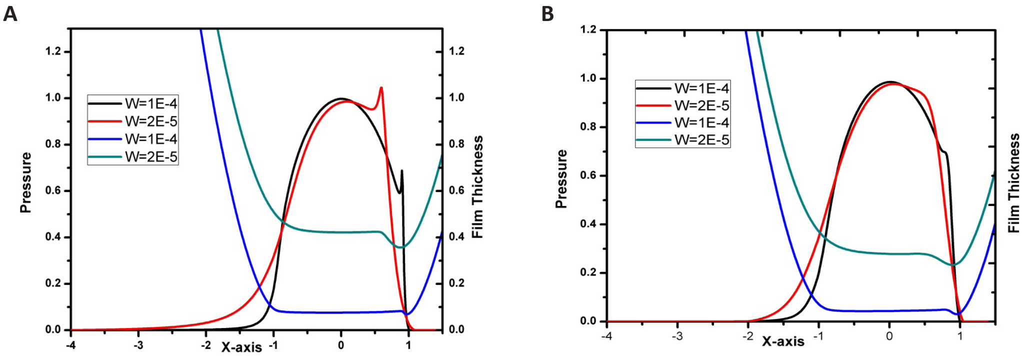

Figure 3. Pressure and film thickness profiles at G_{cr}=300Kpa, U=1E-11, S=0, n=0.4 (A) and at G_{cr}=300Kpa, U=1E-11, S=1, n=0.4 (B) for various values of load.

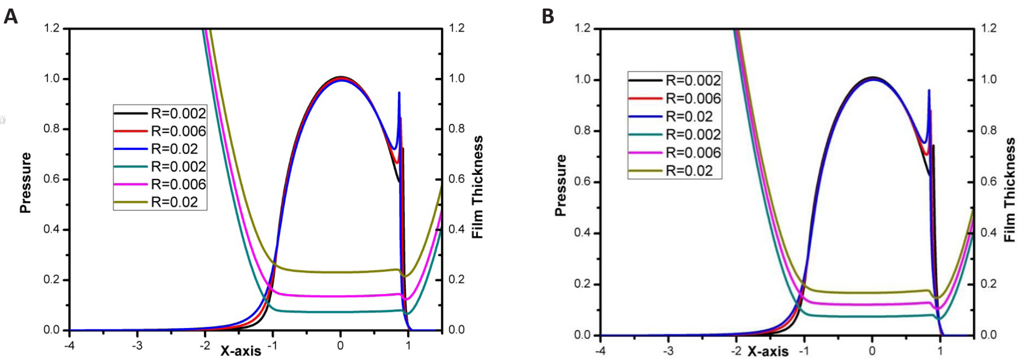

Figure 4. Pressure and film thickness profiles at G_{cr}=50Kpa, u=0.1m/s, S=0 ,n=0.4 ph=1Gpa (A) and at G_{cr}=300Kpa, U=1E-11, S=0, n=0.4 ph=1Gpa (B) for various values of R.

and central film thickness decreased with the increase in load and sliding ratio (Figure 3B).

The pressure and film thickness profiles for speed u=1m/s, ph=1Gpa, Gcr=300Kpa and power index, n=0.4

Table 1. Dolittle and Roelands Predicted Minimum Film Thickness Hmin Versus Calculated Values with Dowson-Higginson Equation, for Different Dimensionless Speed and Load

Sliding/ Rolling Ratio	Speed U	Load W	h_{min}/R[35]	h_{min}/R[2]	h_{min}/R (Present-Method)
S=0	$1x10^{-12}$	$2x10^{-5}$	$2.28x10^{-6}$	$4.2811x10^{-6}$	$3.9280x10^{-6}$
		$1x10^{-4}$	$1.65x10^{-6}$	$3.4729x10^{-6}$	$1.6094x10^{-6}$
S=1	$3x10^{-12}$	$1x10^{-4}$	$2.49x10^{-6}$	$0.7493x10^{-5}$	$1.3175x10^{-6}$
	$1x10^{-11}$		$4.63x10^{-6}$	$1.7406x10^{-5}$	$2.9928x10^{-6}$

and 0.7 with varying radius R are presented in Figure 4A and Figure 4B. The central and minimum film thickness decreased with the decline in the power index, and pressure spike slightly increased with the increase in power index for fixed value of R. With the increase in the value of R, the pressure spike increased and the central and minimum film thickness decreased. Comparisons of numerical and Dowson-Higginson[2] predicted minimum film thickness H_{min} with different dimensionless speed and load are presented in Table 1 and the results are comparable.

Comparing the obtained results of pressure and film thickness profiles using the Free-volume and Roelands model for a viscosity-pressure relationship, it was observed that the pressure spike value for the free volume model was about 20% higher than the Roelands model in the same operating conditions and that real pressure-viscosity behavior predicted by piezo-viscous model yielded a higher viscosity at the pressure area, resulting in a larger length of the central film thickness.

4 CONCLUSION

1) Newton- GMRES method is an effective numerical method to find the solutions to the EHL line contact problem.
2) Pressure spike increases with the increase in speed and moves towards the outlet zone with increasing load.
3) Film thickness decreases with the decline in power index and increase in the value of R.

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Conflicts of Interest
The authors declared no conflict of interest.

Author Contribution
Both authors designed and studied the article, performed the numerical computations and prepared the results in terms of figures and table, and both authors revised the manuscript for important intellectual content and approved the final version.

Abbreviation List
DWT, Discrete wavelet transform
DWTPer, Discrete wavelet transformation with permutation
EHL, Elastohydrodynamic lubrication
GMRES, Generalized minimal residual
KSMs, Krylov subspace methods

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