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Short Communication

# Transmission of Polarized Electrons through Multilayered Spintronics

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## Abstract

**Objectives:** We study transmission of polarized electrons through multilayered spintronics of the form of $n$-cell periodic $(L/R)^n$ and symmetric $(L/R)^n$ multilayers. $L$ and $R$ are layers of ferromagnetic semiconductors or insulators. The multilayers are of arbitrary sizes with layer $L$ of width a and layer $R$ of width $b$. The number of cells $n$ is arbitrary positive integer. For specific examples, we use ferromagnetic semiconductors $Ga_{0.73}Mn_{0.27}N$, $In_{0.92}Fe_{0.08}As$ and $Ga_{0.8}Fe_{0.2}N$ which have different *sd*-exchange energies. For the insulators, we use non-ferromagnetic insulator $A_{10.3}Ga_{0.7}As$, and ferromagnetic insulator $Al_{0.9}Fe_{0.1}Sb$.

**Methods:** For the purpose of the study, we will calculate transmission coefficients of polarized electrons through each multilayer by using the method of transfer matrix for finite periodic systems. Energy dependencies are analytically derived and generalize the transmission through several types of multilayered spintronics, accommodating an arbitrary number of cells and layer widths embedded between ferromagnetic semiconductors. Results will be derived for multilayers with ferromagnetic layers that are the same as or different from the ferromagnetic semiconductors embedded in the multilayer.

**Results:** Various characteristics of transmissions are obtained from low transmissions to high transmissions with sharp peaks about certain energies, depending on the ferromagnetic semiconducting layers and the insulating layers being used for the multilayers. By varying the materials and sizes of the layers one can arrange desired transmissions such as those with high transmissions at resonances.

**Conclusion:** Multilayers can be used as an effective way to obtain polarized electrons with single or multiple selected energies within relatively narrow ranges, enabling their transmission through the multilayer. This is important for transport of polarized electrons in systems of spintronics in general, and also for the study of fundamental physics such as macroscopic quantum phenomena in magnetic spintronics.

**Keywords:** polarized electrons, ferromagnetic semiconductors, multilayers, spintronics

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## 1 INTRODUCTION

Systems of spintronics have been subjects of interests to study various aspects of physics in the last decades. Among these, a few examples include investigations into the effect of a polarized current on the excitations of spin waves in multilayers[1-3], the generation of microwaves in spintronics[4], prospects for antiferromagnetic spintronics[5,6], and a theory on the damping of magnetization in magnetic multilayers[7]. Various materials for charge conduction in semiconducting spintronics have been studied, such as Mn-based systems like (In,Mn)As[8,9], (Ga,Mn)As[10,11], and (Ga,Mn)N[12,13]. Fe-based ferromagnetic semiconductors have been studied in systems like (GaFe)N[14,15] and (In,Fe) As[16]. On the other hand, ferromagnetic insulators for spintronics have been studied in systems of (AlFe)Sb[17] which act as spin-filter tunnel barriers. One can refer to the reference[18] for a review spanning from the theory to the applications of spintronics. More recently, in the subject of fundamental quantum physics in magnetic systems of the order of nanometers, macroscopic quantum phenomena in spintronics have also been investigated both in ferromagnetic spintronics[19,20] and antiferromagnetic spintronics[20,21]. Here, we expend the study of transmission of polarized electrons through multilayers. In this case, the multilayers may involve ferromagnetic semiconductors and non-magnetic or ferromagnetic insulators. Various aspects of transmission of polarized electrons can be studied further, similar to the case of unpolarized electrons in usual electronic systems. This involves starting from regular tunnel junctions and progressing to spin-filtered transmission through tunnel junctions[17]. Therefore, the study will also be important for the investigation of macroscopic quantum phenomena in spintronics.

Of particular interest that we present here is the energy-dependent transmission of polarized electrons through various multilayered spintronics of arbitrary finite size. We will study multilayers with various combinations of ferromagnetic layers that are the same as those embedding the multilayers, as well as with ferromagnetic layers that are different than those embedding the multilayers. The combinations also involve layers of non-ferromagnetic or ferromagnetic insulating layers. In this way, we not only show cases in which combinations of ferromagnetic materials may lead to small transmissions, but we also show how to obtain desirable high transmissions with sharp peaks about certain energies. For concrete examples, we will use ferromagnetic semiconductors $Ga_{0.73}Mn_{0.27}N$, $In_{0.92}Fe_{0.08}As$ and $Ga_{0.8}Fe_{0.2}N$ which have different *sd*-exchange energies. For the insulators, we will use $Al_{0.3}Ga_{0.7}As$ and $Al_{0.9}Fe_{0.1}Sb$. With the variations of ferromagnetic semiconductors and insulators we can then study various multilayers with different characteristics of transmissions of polarized electrons. In addition, we will also show the effect of varying the number of cells and the widths of the layers. In this work, we will show how to obtain high transmission of polarized electrons with relatively sharp peaks at certain energies. In this way we can use the multilayers as spin filters[17] to select various polarized electrons with certain energies which is desirable for transport of polarized electrons in systems of spintronics in general[18]. Such transmission with a precise single energy is crucial in the case of macroscopic quantum phenomena in spintronics where one applies polarized electrons with a specific energy to study transitions in the magnetization of a ferromagnet or the Néel vector of an antiferromagnet, with or without dissipation[19-21]. Here we will calculate transmission coefficients of polarized electrons through various multilayers by using the method of transfer matrix for finite systems[22,23]. We will explain the mechanism and use of multilayered spintronics as spin filters[17].

## 2 THEORY AND METHODS

In the following, we will consider transmissions of polarized electrons through two types of multilayers, i.e., an $n$-cell periodic $(L/R)^n$ and symmetric $(L/R)^nL$ multilayers which are embedded between ferromagnetic ($F$) semiconductors. A layer $L$ has thickness $a$ while a layer $R$ has thickness $b$, and $n$ is an arbitrary positive integer that determines the periodicity and size of the multilayer. $L$ and $R$ are ferromagnetic semiconductor and insulator ($I$), respectively, or vice versa; for example, Figure 1 for multilayer $(L/R)^n$ embedded between ferromagnetic ($F$) semiconductors. We will study transmission of polarized electrons in the tunneling regime for various types of multilayers.

The transmission of polarized electrons through the multilayers will be derived using the method of transfer matrix for finite periodic systems[22,23]. With the method, the wave function $\psi(x_1)$ (that is written as a wave vector) at point $x_1$, and that at $x_2$ are related by a transfer matrix $T(x_1, x_2)$ as $\psi(x_1)=T(x_1, x_2)\psi(x_2)$ in which the transfer matrix possesses a multiplication property as $T(x_1, x_2)=T(x_1, x_i)T(x_i, x_2)$. The laws of physics, including the time reversal symmetry, that govern the multilayer and the boundary conditions satisfied by the wave function throughout the multilayer are taken into account in the transfer matrix. The physical parameters of the multilayer, such as the size and type of each layer, are also taken into account in the transfer matrix. For the periodic part within the multilayer, the transfer matrix is determined in terms of the parameters of a unit cell of the multilayer. In this case, of particular importance is to obtain the matrix elements of a unit-cell transfer matrix $T(x_0, x_0+\ell)$ (where $x_0$ can be arbitrarily chosen, and $\ell=a+b$ is the periodicity of the multilayer), such as the matrix element $\alpha=T_{11}(x_0, x_0+\ell)$-element and the matrix element $\beta=T_{12}(x_0, x_0+\ell)$-element of the unit cell (with the other elements may be related to them). By the multiplication property of the transfer matrix, the elements of the total transfer matrix for the complete multilayer will then be determined in terms of those matrix elements of

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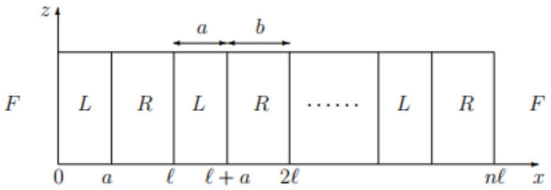


**Figure 1. *xz*-cross section of an *n*-cell periodic layers (*L/R*)$^n$ embedded between ferromagnetic (*F*) semiconductors at *x*<0 and *x*>*nℓ*.** A layer *L* has width *a* while a layer *R* has width *b* so that the periodicity *ℓ*=*a*+*b*.

the unit-cell transfer matrix. The wave function and the transport properties of the multilayer can be obtained[22,23] and the space-time evolution of the wave throughout the system can also be determined and studied[24,25].

To start with an example of our study, we consider a periodic n-cell multilayer $(L/R)^n$ where $L$ is a ferromagnetic ($F$) semiconductor while $R$ is an insulator ($I$). We assume themultilayer to be located at $0 \leq x \leq n\ell$ and embedded between ferromagnetic semiconductors at $x<0$ and $x>n\ell$ as in Figure 1.

Let us consider an incoming polarized electron wave $\psi(x)=\exp(ikx)$ from the left ($x<0$) of the multilayer. The incoming wave may be assumed to make an angle $\theta$ to the normal ($x$ axis) of the multilayer with $k=|\mathrm{k}|\cos\theta$ and $|k| = \sqrt{2\mathrm{m}(\mathrm{E}+\mathrm{V}_e)/\hbar^2}$ where $m$ is the effective mass of an electron, $E$ is the energy, and $V_e$ is the average $sd$-exchange energy in the ferromagnetic semiconductor. $\hbar = h/2\pi$ with Planck constant $h$. By conserving the in-plane momenta, the wave will then be transmitted through the multilayer as $\psi(x)= \mathcal{T} \exp(ikx)$ at $x>n\ell$ with $\mathcal{T}$ the transmission amplitude.

With the method of transfer matrix for finite periodic systems[22,23], the matrix elements $\alpha=T_{11}(0, \ell)$ and $\beta=T_{12}(0, \ell)$ of the unit cell for the system considered are, respectively,

$$\alpha = \frac{e^{\mathrm{i}ka}}{2i}\left(\frac{q}{k}-\frac{k}{q}\right)\sinh(qb) + e^{ika}\cosh(qb) \quad (1)$$

$$\beta = \frac{e^{-\mathrm{i}ka}}{2i}\left(\frac{q}{k}+\frac{k}{q}\right)\sinh(qb) \quad (2)$$

Here, $\mathrm{q} = \sqrt{\kappa^2\text{-}\mathrm{k}^2\sin^2\theta}$ where $\kappa = \sqrt{2\mathrm{m}_\mathrm{I}\,(V_\mathrm{I}\text{-E})/\hbar^2}$ with $m_I$ the effective mass of an electron and $V_I$ the barrier height in the insulating layers. Furthermore, with the method of transfer matrix[22,23], the transmission amplitude $\mathcal{T}$ is given by

$$\mathcal{T} = \frac{1}{\alpha_\mathrm{T}^*}\mathrm{e}^{-ik\ell} \quad (3)$$

Here, the matrix element $\alpha_T=T_{11}(0, n\ell)$-element of the total transfer matrix of the multilayer. In this case,

$$\alpha_\mathrm{T} = \mathrm{U_n}(\alpha_\mathrm{R}) \text{ - } \alpha^*\mathrm{U_{n\text{-}1}}(\alpha_\mathrm{R}) \quad (4)$$

where $U_n(x)$ are the Chebyshev polynomials of the second type[26], and $\alpha_R$ is the real part of $\alpha$. One finds the transmission coefficient $c_T=|\mathcal{T}|^2$ as

$$C_T = \frac{1}{1+|\beta|^2\mathrm{U}_{n-1}^2(\alpha_\mathrm{R})} = \frac{1}{1+\frac{1}{4}\left[\frac{k}{q}+\frac{q}{k}\right]^2\sinh^2(qb)U_{n-1}^2(\alpha_\mathrm{R})} \quad (5)$$

This results in transmission coefficients without a true gap as a function of energy. The transmission coefficient has resonances when $U_{n-1}(\alpha_R)=0$, so that

$$\cos(\mathrm{ka})\cosh(\mathrm{qb}) + \frac{1}{2}\left[\frac{q}{k}-\frac{k}{q}\right]\sin(ka)\sinh(qb) = \cos\left(\frac{v\pi}{n}\right) \quad (6)$$

with $v=0, \pm1...$ The last expression shows that a resonance for a certain number $n$ of cells, can also be a resonance for other numbers of cells, such as resonances at energies $E_{v,n}$ as $E_{1,3}=E_{2,6}=E_{3,9}$, etc.

Analogously, for symmetric multilayer $(L/R)^nL$ the results for the matrix element $\alpha_T$ and the transmission coefficient $c_T$ are also obtained in a similar fashion (see below).

## 3 RESULTS AND DISCUSSION

To illustrate our results, as the first example we consider multilayer $(L/R)^n$ with ferromagnetic layer $L=Ga_{0.73}Mn_{0.27}N$ that has $sd$-exchange energy $V_e=0.23$eV, and non-ferromagnetic insulating layer $R=Al_{0.3}Ga_{0.7}As$ that has barrier height $V_I= 0.23$eV.

To see an immediate effect of the exchange energy, by utilizing the properties of resonances we show in Figure 2 the transmission coefficient when the number of cells $n=2$, for polarized electrons (red curve with single peak) and unpolarized electrons (blue curve with four peaks). This shows the advantage of using multilayer $(L/R)^2$ with polarized electrons as to produce precise transmission of polarized electrons with only one specific energy (red curve) with relatively small width; the width that can be made thinner by having a thicker insulating layer $b$ for a fixed ferromagnetic layer width $a$ (see below).

We also show in Figure 3 transmission coefficients when $a=100$Å and $a=200$Å for fixed $b=30$Å, which shows the importance of using a certain width of the ferromagnetic layers in multilayer $(L/R)^2$ when using polarized electrons such as to produce precise transmission with a specific energy (red curve). Furthermore, Figures 2 and 3 show the effect of $sd$-exchange as a spin-filter mechanism to transmit polarized electrons with high transmission about resonances and to suppress those others with low transmission. Therefore, with the properties of resonances, the use of multilayer with $n=2$ provides a simple way to obtain a spin-filter with only a small number of resonances, such as shown in Figure 3 with only a single-peaked (red curve) polarized electrons with resonance energy at

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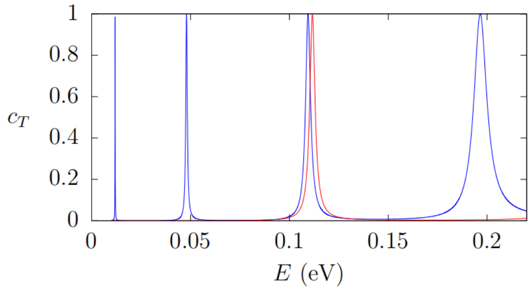


**Figure 2. Transmission coefficients of polarized and unpolarized electrons of normal incidence as a function of energy *E* through multilayer ($Ga_{0.73}Mn_{0.27}N/Al_{0.3}Ga_{0.7}As)^2$ (as Figure 1 with *n*=2).** Red curve is for polarized electrons while blue curve is for unpolarized electrons. We have used $m=0.2m_0$ with $m_0$ the mass of free electron, $V_e$=0.23eV, $a$=100Å, $b$=30Å, $V_I$=0.23eV, and $m_I=0.1m_0$ in $Al_{0.3}Ga_{0.7}As$ layers.

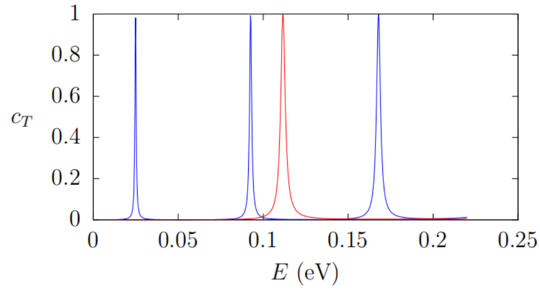


**Figure 3. Transmission coefficients of polarized electrons of normal incidence as a function of energy *E* through multilayer ($Ga_{0.73}Mn_{0.27}N/Al_{0.3}Ga_{0.7}As)^2$ (as Figure 1 with *n*=2).** Red curve uses width $a$=100Å while blue curve uses $a$=200Å. Other parameters are as in Figure 2.

$E$=0.1116eV when using $a$=100Å and $b$=30Å. The width about resonance can be made thinner by taking a thicker insulating width $b$ such as $b$=40Å which results in the resonance energy a bit shifted to $E$=0.1106eV.

For a comparison, to show the energy dependence for a bigger number n of cells, we have plotted in Figure 4 the transmission coefficient for $n$=6 which has resonances compared to that for $n$=1 with much lower transmission without resonance. This shows how the number of cells $n$>2 can produce multiple high transmission with precise energies.

Another example, for a higher energy regime, we consider a multilayer with ferromagnetic $Al_{0.9}Fe_{0.1}Sb$ as the insulating layers[17]. Figure 5 compares transmission coefficients through multilayer ($Ga_{0.73}Mn_{0.27}N/Al_{0.3}Ga_{0.7}As)^2$ (blue curve, see also Figure 2) and multilayer ($In_{0.92}Fe_{0.08}As/Al_{0.9}Fe_{0.1}Sb)^2$ (red curve). This indicates how the ferromagnetic insulating $Al_{0.9}Fe_{0.1}Sb$ is effective as a fine-tuned spin-filter tunnel barrier for transmission of polarized electrons with specific relatively high energy.

Figure 6 describes transmissions through multilayer ($In_{0.92}Fe_{0.08}As/Al_{0.9}Fe_{0.1}Sb)^n$ when $n$=2 and $n$=6. Interestingly, Figures 5 and 6 show that only polarized electrons with certain energies within relatively small widths that are transmitted through the multilayers. The red curve in Figure 5 for $n$=2 shows a sharp peak at $E$=0.5024eV, while Figure 6 for $n$=6 (blue curve) shows sharp peaks at several energies. This indicates how the insulating ferromagnetic $Al_{0.9}Fe_{0.1}Sb$ is effective as a fine-tuned spin-filter tunnel barrier as suggested in the reference[17]. Figure 6, on the other hand, shows the case of $n$=6, which shows how the ferromagnetic insulating $Al_{0.9}Fe_{0.1}Sb$ is effective as a fine-tuned spin-filter tunnel barrier for transmission of polarized electrons of various energies (blue curve). By utilizing the properties of resonances, one may then use a multilayer with $n$=2 to transmit polarized electrons with a selected energy of resonance, by varying the width $a$ of the ferromagnetic layers for fixed width $b$ of the insulating layers, or vice versa.

Consider now multilayer $(L/R)^n$ where $L$ is an insulator while $R$ is ferromagnetic ($F'$) semiconductor that is different than the ferromagnetic $F$ embedding the multilayer. An example is a multilayer with $F'=Ga_{0.8}Fe_{0.2}N$ which has *sd*-exchange energy $V'_e$=0.1eV[14,15], and insulating layers $Al_{0.3}Ga_{0.7}As$ with barrier height $V_I$=0.23eV. We may take the embedding semiconductors $F=In_{0.92}Fe_{0.08}As$[16]. In this case, with incomingwave number $k=|k|\cos\theta$, the wave number in $F'$ layers is $|k'|=\sqrt{2m'(E+V'_e)/\hbar^2}$ while in the insulating ($I$) layers $q$ is as before. Here we have a unit cell with the matrix elements $\alpha$ and $\beta$ given, respectively, by

$$\alpha = e^{ik'b}[\cosh(qa) - \frac{i}{2}(\frac{q}{k'} - \frac{k'}{q})\sinh(qa)] \quad (7)$$

$$\beta = -\frac{i}{2}e^{ik'b}(\frac{q}{k'} + \frac{k'}{q})\sinh(qa) \quad (8)$$

In this case, the $T_{11}$-element $\alpha_T$ of the total transfer matrix $T(0, n\ell)$ for the transmission amplitude is $\mathcal{T}$ now

$$\begin{aligned} \alpha_T = & \frac{ie^{ik'b}}{4kqk'}[(-k-k')\alpha_{n-1} + (-k+k')\beta^*_{n-1}] \\ & \times[(-kk'+q^2)\sinh(qa) + i(k+k')q\cosh(qa)] \\ & + \frac{ie^{-ik'b}}{4kqk'}[(-k-k')\beta_{n-1} + (-k+k')\alpha^*_{n-1}] \\ & \times[(-kk'+q^2)\sinh(qa) + i(k-k')\cosh(2qa)] \end{aligned} \quad (9)$$

Here, we have

$$\alpha_n = U_n(\alpha_R) - \alpha^* U_{n-1}(\alpha_R) \quad (10)$$

$$\alpha_n = U_n(\alpha_R) - \alpha^* U_{n-1}(\alpha_R) \quad (11)$$

Figure 7 plots the transmission coefficient through multilayers $(L/R)^n$ for $n$=2 and $n$=4 when $L$ is insulator $Al_{0.3}Ga_{0.7}As$, and $R$ is ferromagnetic semiconductor $F'=Ga_{0.8}Fe_{0.2}N$ where the multilayer is embedded between ferromagnetic semiconductor $F=In_{0.92}Fe_{0.08}As$.

The figure shows relatively broaden curve with low transmissions which may not be convenient for spin-filter spintronic systems, unless low transmissions are the ones needed. The figure in general shows relatively

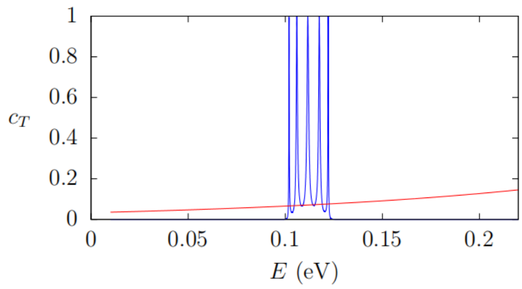


**Figure 4. Transmission coefficient of polarized electrons of normal incidence as a function of energy $E$ through multilayer $(Ga_{0.73}Mn_{0.27}N/Al_{0.3}Ga_{0.7}As)^n$ with $n$=1 (red curve) and $n$=6 (blue curve).** Other parameters are as in Figure 2.

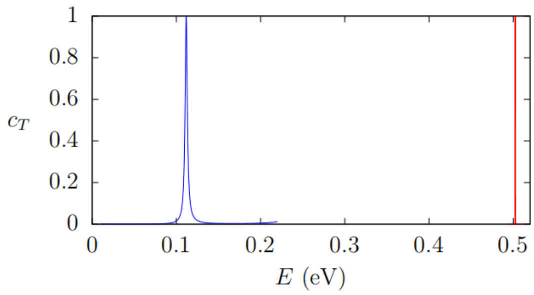


**Figure 5. Transmission coefficient of polarized electrons of normal incidence as a function of energy $E$.** Blue curve is through $(Ga_{0.73}Mn_{0.27}N/Al_{0.3}Ga_{0.7}As)^2$ with non-ferromagnetic insulating layers (for 0<$E$<0.23eV) while red curve is through multilayer $(In_{0.92}Fe_{0.08}As/Al_{0.9}Fe_{0.1}Sb)^2$ with ferromagnetic insulating layers.

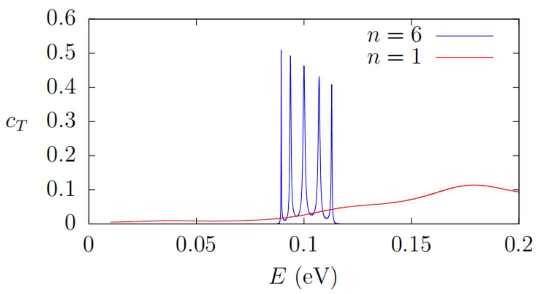


**Figure 8. Transmission coefficient of polarized electrons of normal incidence as a function of energy $E$ through multilayer $(F'/I)^nF'$=$(Ga_{0.8}Fe_{0.2}N/Al_{0.3}Ga_{0.7}As)^nGa_{0.8}Fe_{0.2}N$ for $n$=1 (red curve) and $n$=6 (blue curve), embedded between semiconductors $F$=$In_{0.92}Fe_{0.08}As$.** We have used $V_e$=2.8eV, $m$=0.11$m_0$ in $In_{0.92}Fe_{0.08}As$, $V'_e$=0.1eV, $m'_F$=0.2$m_0$ in $Ga_{0.8}Fe_{0.2}N$; $a$=100Å, $b$=30Å, $V_I$=0.23eV, and $m_I$=0.1$m_0$ in $Al_{0.3}Ga_{0.7}As$ layers.

lower transmissions compared to those in the previous cases with resonances when the ferromagnets inside the multilayers are the same as those embedding the multilayer. This shows how the use of ferromagnetic layers $F'$ that are different than those $F$ embedding the multilayer may be of disadvantage to produce transmitted polarized electrons with specific energies of small widths.

To complete our examples for comparisons we now consider symmetric multilayer $(L/R)^nL$ with $L$ ferromagnetic

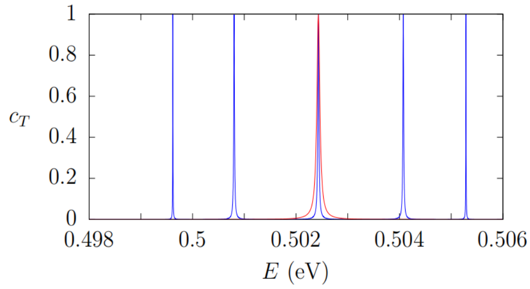


**Figure 6. Transmission coefficient of polarized electrons of normal incidence as a function of energy $E$ through multilayer $(In_{0.92}Fe_{0.08}As/Al_{0.9}Fe_{0.1}Sb)^6$ (blue curve) when $n$=6, and the case of $n$=2 (red curve).** We have used $V_e$=2.8eV, $m$=0.11$m_0$, $a$=100Å, and $b$=30Å, $m_I$=0.11$m_0$, $V_I$=1.15eV in $Al_{0.9}Fe_{0.1}Sb$ layers.

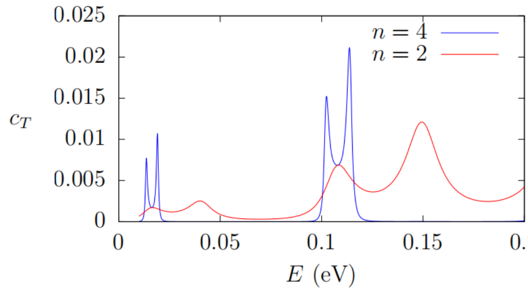


**Figure 7. Transmission coefficient of polarized electrons of normal incidence as a function of energy $E$ through multilayer $(Al_{0.3}Ga_{0.7}As/Ga_{0.8}Fe_{0.2}N)^n$ with $n$=2 (red curve) and $n$=4 (blue curve), embedded between ferromagnetic semiconductor $In_{0.92}Fe_{0.08}As$.** We have used $V_e$=2.8eV, $m$=0.11$m_0$ in $F$, $V_e'$=0.1eV, $m'_F$=0.2$m_0$ in $F'$, $a$=30Å, $b$=100Å, $V_I$=0.23eV, and $m_I$=0.1$m_0$ in $Al_{0.3}Ga_{0.7}As$ layers.

semiconductors $F'$ different than the ferromagnetic semiconductors $F$ embedding the multilayer. An example is multilayer with $L$=$Ga_{0.8}Fe_{0.2}N$, and insulator $R$=$Al_{0.3}Ga_{0.7}As$ with barrier height $V_I$=0.23eV, while the embedding ferromagnetic semiconductors are $F$=$In_{0.92}Fe_{0.08}As$. The matrix elements $\alpha$ and $\beta$ are as Equations (1) and (2), respectively, but with $k \longrightarrow k'$. The matrix element $\alpha_T = T_{11}(0, n\ell + a)$ of the total transfer matrix for the transmission amplitude $\mathcal{T}$ becomes

$$\begin{aligned}\alpha_T &= \frac{e^{ik'a}}{4}(1+\frac{k'}{k})\left[(1+\frac{k'}{k})\alpha_n + (1-\frac{k'}{k})\beta_n\right] \\ &+ \frac{e^{-ik'a}}{4}(1-\frac{k'}{k})\left[(1+\frac{k}{k'})\beta_n^* + (1-\frac{k}{k'})\alpha_n^*\right]\end{aligned} \quad (12)$$

The transmission coefficient through multilayers $(Ga_{0.8}Fe_{0.2}N/Al_{0.3}Ga_{0.7}As)^nGa_{0.8}Fe_{0.2}N$ for $n$=1, 6 is shown in Figure 8. The figure shows that the use of ferromagnetic layers $F'$ different than those $F$ embedding the multilayer leads to low transmission of polarized electrons as compared to those when $F'$=$F$. The figure also shows relatively lower transmissions compared to those in the previous cases.

## 4 CONCLUSION

We have calculated analytically exact transmission coefficients of polarized electrons through finite multilayered spintronics that consist of ferromagnetic semiconductors and insulating layers. Variations on the ferromagnetic semiconductors (of different *sd*-exchanges) and insulators of the layers allow us to study different characteristics of transmission of polarized electrons through various multilayers. By varying the number of cells and the thicknesses of the layers one can use multilayers as spin filters to select various polarized electrons with high transmissions at certain energies and suppressing the others with low transmissions. The use of ferromagnetic semiconductors with high *sd*-exchange energy and ferromagnetic insulating layers shows an effective way to obtain polarized electrons with selected energies within relatively small widths to be transmitted through the multilayer, so that the multilayer behaves as a spin-filter for polarized electrons with single or multiple precise energies depending on the number of cells n. This is desirable for transport of polarized electrons not only in systems of spintronics in general, but also especially important for the study of fundamental physics such as macroscopic quantum phenomena in ferromagnetic and antiferromagnetic spintronics where one uses polarized electrons with desirably precise energy to study transition of the magnetization in ferromagnetic spintronics and the Néel vector of antiferromagnetic spintronics with or without dissipation.

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**Conflicts of Interest**
The author declared no conflict of interest.

**Author Contribution**
Simanjuntak HP solely contributed to the manuscript and approved the final version.

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